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**Making sense of mathematical  
language in a primary classroom**

**DOCTOR OF EDUCATION (EdD)**

28<sup>th</sup> September 2001

**Abstract: Making sense of mathematical language in a primary classroom**

This dissertation describes a classroom-based research project on the language of primary school mathematics as used in three mathematics lessons and eight structured interventions with children. My aim is to analyse the classroom dialogue and consider the effectiveness of the participants' communication processes as they try to share their understanding of meanings of mathematical language. Literature from several research disciplines informs the analysis, although my prime interest is communication and thought processes. Methods were refined during a pilot project for Stage 1 of the EdD in the same classroom. My research shows that in some situations, mathematical language is far from precise in meaning, and the communicative processes used to make it potentially shareable are often tentative and transient according to the situation. In particular, I question the idea of setting mathematics into everyday contexts in order to improve communicative relevance, because children bring their own previous knowledge and experience to the interpretation of each situation. My analysis highlights that reference to everyday contexts might not be effective in communicating the meaning of probabilistic language. Part of the difficulty lies with probabilistic words also having everyday meanings, but the main difficulty is that few life events can be given probabilities such as 'certain' and 'even chance'. Gestures and pictorial images are also influential when trying to communicate one's understanding of meaning. A tentative conclusion is that referring to proportional relationships involving number, rather than real-life events might provide opportunity for more effective communication of probabilistic meanings. Teachers need to be aware that the language of mathematics is not always precise and that pictorial images and gesture have a powerful effect on the development of a shared understanding of meaning.

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This dissertation describes a classroom-based research project on the language of primary school mathematics as used in three mathematics lessons and eight structured interventions with children. My aim is to analyse the classroom dialogue and consider the effectiveness of the participants' communication processes as they try to share their understanding of meanings of mathematical language. Literature from several research disciplines informs the analysis, although my prime interest is communication and thought processes. Methods were refined during a pilot project for Stage 1 of the EdD in the same classroom. My research shows that in some situations, mathematical language is far from precise in meaning, and the communicative processes used to make it potentially shareable are often tentative and transient according to the situation. In particular, I question the idea of setting mathematics into everyday contexts in order to improve communicative relevance, because children bring their own previous knowledge and experience to the interpretation of each situation. *My analysis highlights that reference to everyday contexts might not be effective in communicating the meaning of probabilistic language.* Part of the difficulty lies with probabilistic words also having everyday meanings, but the main difficulty is that few life events can be given probabilities such as 'certain' and 'even chance'. Gestures and pictorial images are also influential when trying to communicate one's understanding of meaning. A tentative conclusion is that referring to proportional relationships involving number, rather than real-life events might provide opportunity for more effective communication of probabilistic meanings. Teachers need to be aware that the language of mathematics is not always precise and that pictorial images and gesture have a powerful effect on the development of a shared understanding of meaning.

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## **Chapter 1: Introduction**

The requirements of Stage 2 of the Open University EdD Language and Literacy line of study shaped the nature and scope of the research I describe. The research itself is concerned with the language of mathematical activities in a primary classroom. My background is that of a primary education specialist, with most recent work experience in Initial Teacher Training and *Continuing Professional Development*, including in- class support and demonstration lessons. It is important to me that I research an aspect of teaching and learning that has prompted much personal reflection and professional development, due to experiences and observations made over a number of years. My purpose in completing this work is to further inform my own practice, that of my colleagues in training institutions, and to add to the growing body of research that explores the complexities of primary school classroom situations from various perspectives. I view my EdD research as a taught and valid development, firmly based in classroom practice and highly relevant to primary practitioners, in addition to being worthy of publication.

This chapter sets out some of the initial influences on the choice and development of my research focus. First, I identify classroom experiences and initial literature sources. I then consider recent changes in initial teacher training, continuing professional development and school curricula that affect the direction of my work, especially the idea of ‘precision’ in the use of mathematical language. I also define specific phrases used, set out the research questions and parameters for the research focus, before briefly outlining subsequent chapter content and structure.

### **My interest in mathematical language**

Several years ago, I observed classroom situations where children interpreted word meanings differently from the teacher e.g. when a student teacher stressed the word ‘half’ while bisecting a shape on the board a child thought the *line* was ‘half’. In another classroom, a teacher talked about the sides ‘growing up’ from the rectangular base, when building a cuboid. One

child interpreted 'cuboid' as a grown-up word for 'rectangle'. In each example, previous knowledge and experience determined the child's response to the teacher's use of the mathematical word (Halliday, 1978). It is also important to note that in both examples the teachers were actively pointing, or manipulating materials while talking. Their movements, and the diagrams or resources used also affected the child's response. The response was not just to the spoken word. Such observations prompted me to question the effectiveness of dialogue during mathematics lessons, in relation to teachers and children developing a shared mathematical vocabulary.

The report 'Mathematics Counts' (Cockcroft, 1982), commonly referred to as the Cockcroft Report, is the only national specially commissioned report on mathematics. It summarises a review of relevant research projects from the 1970's and early 1980's. The purpose was to identify the causes for concern expressed by government, educators and employers about national levels of achievement. Much of the research reviewed was from secondary schools, and not necessarily directly transferable to the primary sector. However, it is important to note the issues it raised about mathematical language, especially those concerning the precision of mathematical language and the need to explain mathematics in words. One review included amongst others, three significant aspects for my research:

1. Language and the formation of mathematical concepts
2. Oral language in the mathematical classroom, and
3. Mathematical terminology, symbolism and linguistic structure

(Bell, Costello and Küchemann, 1983, page 273)

Bell *et al.* (1983) concluded that oral dialogue is an essential part of the process of learning mathematics, and that children are disadvantaged if they do not have a mathematical vocabulary. However, they made little comment about the nature of mathematics classroom dialogue except to compare 'teacher-directed' classes with 'child-centred' classes. For example, a child-centred approach leads to use of unorthodox language that can either be problematical or provide new opportunities. Interpretation and explanation

of mathematics through language were preferable to learning a mathematical vocabulary without understanding. Bell *et al.* (1983) clearly stated that they did not support the notion that mathematics is a precise and unambiguous language, but that it is a body of activity and knowledge often enacted in language (Bell, *et al.* 1983). The Cockcroft Report emphasised the enactment through language and therefore added strong support to previous recommendations by Her Majesty's Inspectorate (Department of Education and Science [DES], 1979) that discussion during mathematics lessons was important, and that children ought to learn how to communicate their mathematics using mathematical vocabulary where appropriate.

Guidance in the Cockcroft Report about how children best learn specific mathematical language is vague and implies that providing the opportunity to read, write and speak mathematics ensures learning of the language. In my experience, the influence of the report on actual primary classroom practice was minimal. Primary teachers with an interest in developing their mathematics teaching skills began to use investigative approaches and encouraged group discussion during lessons. They recognised the importance of developing explanation and process skills, but this development was sometimes unstructured and at the expense of learning a mathematical vocabulary. A few focused on teaching the children a mathematics register in a structured approach linked to understanding of the mathematics involved (Halliday, 1978). However, until recently the majority of primary school children have failed to develop a secure mathematical vocabulary by the age of eleven (Qualifications and Curriculum Authority [QCA], 2000). QCA's analysis of children's responses to Key Stage 2 national tests in 2000 (Qualifications and Curriculum Authority [QCA], 2001) specifically highlights children's difficulties with explaining their reasoning about probabilistic events. In particular, they identify a difficulty with providing clear, unambiguous explanations for specific outcomes.

Another issue raised in the Cockcroft Report was that we use words differently in mathematics lessons than they are in other situations. Bell *et al.* (1983) examined research by Rothery (1980) whose findings are more

fully discussed in Dickson, Brown and Gibson (1984 *1993 reprint*) which is an accepted compilation of research relevant to those involved in initial teacher training. Rothery (1980) categorised mathematical words into three broad sets. He identified that some words are not specifically mathematical, but we use them for a mathematical purpose e.g. 'certain' to express probability. Such words have the same, or roughly the same meaning in both mathematical and everyday contexts. Confusion arises because we do not always use such words in a precise manner in everyday conversation. For example, on mislaying my keys I will say, "I am certain I left them on the table," when I am not certain at all! Others, such as names of geometrical shapes and numerals, are clearly mathematical. They can, in some circumstances, have precise meanings, e.g. parallelogram, two. However, both these examples can be problematic for children. One might define parallelogram in various ways and children struggle with defining such a word even at secondary school as found by Otterburn and Nicholson, (1976) (Bell *et al.* 1983; Dickson *et al.* 1984 *1993 reprint*). 'Two' means the number of objects in a set, the number of an object in a count, or a place on a number line. The digit '2' is not always 'two' when combined with other digits. It becomes 'twe' in twelve and twenty with its roots in the old English word 'twene' meaning 'two'. In '20' it combines with the zero, but in 22 it is named 'twenty' in its own right in the tens position, and 'two' in the units position. Younger pupils must struggle to make sense of the various ideas the digit represents. Other mathematical words have different meanings in the everyday context e.g. volume. Children have to learn that the mathematics lesson about volume uses the word with a different meaning than in a science lesson on sound, during a music lesson, or when adjusting controls on the television. There is general agreement among researchers of mathematical language that difficulty with learning the vocabulary is due to the potential range of meanings of each word or phrase. This has been well researched and documented (Pimm, 1987; Durkin and Shire 1991).

### **The influence of recent national initiatives**

Recent British government initiatives have raised my interest in the idea of

'precision' in the use of mathematical language, but documentation is unclear about what exactly 'precise' use of mathematical language means. The imposed National Curriculum for Mathematics in initial teacher education includes student teachers learning to use a precise mathematical language, and to teach pupils the necessary mathematical terminology (Department for Education and Employment [DfEE], 1998). There seems to be an assumption that each mathematical word has only one clear definition and function if one accepts that 'precise' means 'exactly defined and stated' (Merriam-Webster, 1996). One only has to compare a range of mathematical dictionaries or texts to discover that different definitions exist for the same word. For example, consider two definitions of the word 'area'.

"The area is the size of a surface. The surface may be plane (flat) or curved," (Abdelnoor J.R.E. 1989, p 12),

"Area is a measure of the amount of two dimensional space inside a boundary", (Haylock, D. 1995, p. 188).

The first definition provides something tangible for pupils to identify with, in the form of a surface. Pupils can run their hands over it, they recognise surface in a range of contexts e.g. a table, skin, water; it is measurable in a variety of ways. The second however, suggests that the area is a space. If one perceives space in three dimensions the idea of two-dimensional space seems strange. When area is measured it must have a boundary, but it is difficult to equate this with measuring the area of skin on one's body. Neither definition is incorrect, but neither provides the complete picture. There are dangers in defining and using mathematical words too precisely. While some mathematicians will state that mathematics is a concise and unambiguous language, others will question whether there can ever be any such notion in the learning of mathematics (Rowland, 2000). Rowland observed vagueness in the nature of mathematical discourse, and suggests that this is an essential feature i.e. there is sometimes a need to be tentative and less 'precise'. He views vagueness in language as a versatile device by which speakers can make mathematical assertions with as much precision as

the situation warrants.

Those who view mathematics as a precise language most likely hold an absolutist philosophy about mathematics, i.e. that mathematical knowledge is absolute truth. The two ideas seem to fit well together. Alternatively, those who follow a fallibilist philosophy believe that mathematical knowledge is corrigible and open to revision (Ernest, 1991). However the fallibilist view does not include thoughts about the means by which such knowledge may be questioned i.e. language. Ernest (1991) describes and analyses the contribution of different philosophies to mathematics education and proposes 'social constructivism'. He states that social constructivists believe mathematical knowledge is a social construction. We construct mathematical knowledge through linguistic knowledge (Ernest, 1991). This linguistic knowledge, and hence the mathematical knowledge is viewed as fluid and able to be reconstructed with each new experience. Ernest (1994) discusses different researchers perspectives on the nature of social constructivism and the different importance they attach to individual construction of meaning and the social negotiation of meaning. Whatever the emphasis, a social constructivist perspective places importance on both the social processes and the individual sense making. The notion of a precise mathematical language appears to ignore the need for learners to reconstruct meaning for themselves, through sharing their ideas with others. A tension therefore exists between the idea of a precise language, and the multi-faceted nature of the mathematical concept that the language describes and explains (Pirie, 1997).

One might question the philosophy behind the National Numeracy Strategy, as it appears to require 'precision' in mathematical language, yet flexibility in conceptual development. The Framework for Teaching Mathematics from Reception to Year 6 (DfEE, 1999a) supports the requirement to teach pupils a mathematical vocabulary. It introduces new mathematical words for each year group and states that pupils should "be able to explain their methods and reasoning using correct mathematical words" (DfEE, 1999a, pp. 4-5). It also claims that better standards of numeracy occur when teachers use and

expect pupils to use correct mathematical vocabulary. The Office for Standards in Education (Ofsted) also claim that a useful indicator of a pupils understanding, knowledge and skill of mathematics is their ability to explain their methods and reasoning using correct mathematical language (Ofsted, 1999). However, there is little real guidance as to what constitutes 'correct' use of such vocabulary. In fact, words are included in the Framework about which mathematicians debate, e.g. the use of 'times' in multiplication. The accompanying vocabulary book (DfEE, 1999b, p. 2) provides a little, oversimplistic guidance to teachers about how pupils develop an understanding of mathematical vocabulary, but fails to provide teachers with the definitions of words. The expectation is that teachers and pupils use dictionaries to help them clarify word meanings. Alternatively, the meaning of a word becomes explicit through activity. The result is that teachers use a range of vocabulary without necessarily linking it to conceptual understanding. Pupils learn that different words mean the same thing, when they represent different mathematical structures or contexts e.g. 'difference' and 'take away' in subtraction. In a practical situation with items to represent numbers a 'difference' is a comparison of two sets, while a 'take away' is the removal of some items from one original set. The resulting symbolic statement might be the same. One argument is that knowledge of the underlying conceptual structure attached to the word is irrelevant when one can identify the *symbolic* language producing the correct answer. A dictionary definition of 'difference' is that it is the result of subtracting one number from another (Jones and Clamp, 1991). Older students also use the term in 'difference of two squares' and 'symmetric difference of sets'. The notion of 'precision' in the use of mathematical words is therefore questionable because lexical ambiguity exists in a variety of forms (Pimm, 1987, Durkin and Shire, 1991). Precision in the use of a particular word or phrase therefore suits a particular mathematical situation. However, if we take the word 'difference' it does mean *similar* things in these different mathematical situations, just as the word *mouth* applies to the opening in someone's face, or the opening of a river into the sea (Durkin and Shire, 1991).



The symbolic form of mathematical language is less ambiguous than the linguistic form in relation to children describing and explaining mathematics. However, part of my research focus is to question the desirability of using linguistic forms in 'precise' ways, especially when the teacher or pupils often use specific contextual references in order to exemplify meaning. A 'contextual reference' is a spoken or recorded reference to an actual *physical situation* or a *mental representation* of such, in which the child or teacher perceives application of the mathematical word. Pupils often construct knowledge of mathematical language through 'everyday' situations set by the teacher. The use of a word within an 'everyday' context might limit the child's application of the word in other contexts. The contextual reference used by the teacher may not be within the experiences of the child, and so the child constructs a different meaning of the word. Even the relevance of a simple shopping activity is questionable for children who go shopping at the supermarket where parents use a cash card rather than money. I am not suggesting that such 'everyday' contexts are not useful, only that children might not attach the same meaning to the situational context presented to them by the teacher. My early explorations showed that primary school children referred to 'real-life' contexts when trying to explain mathematical words. Such contextual references that children use to describe mathematical words are of particular interest to me. My aim is to discover the ways that teachers and children communicate meaning of specific words and phrases used within mathematics lessons. My intention is to analyse the ways in which mathematical words emerge in classroom discourse, in relation to the children's developing knowledge and understanding of the language and its use. I will focus on the examination of the 'contextual references' in dialogue and exploration of how they affect the communication process. The main conceptual strand involves the location of meaning, in relation to communicating that meaning to others. Linguistic research highlights the importance of teachers and children developing a shared understanding of meaning (Edwards and Mercer, 1987). However, it seems likely that 'contextual references', perceived or real, have an effect on the development of such shared understanding of meaning. Little recent research explores such 'contextual references' and the way that

miscommunication occurs through participants' inability to create joint meaning of a common reference. This research is unique and so adds to the extensive body of research by both mathematics educators and linguists.

The phrase 'even chance', introduced by the teacher during a lesson on probability is one chosen for analysis because the teacher spent a whole lesson on it, and the children provided a variety of explanations for it. Many school mathematics texts use the term 'even chance' to describe a prediction that places an event halfway along a probability scale of impossible to certain e.g. Numeracy Focus 5 (Ebbutt and Askew, (eds.) 2000). Primary school teachers might also use 'even chance' to describe the possible outcome of an event with two equally likely outcomes as in Collins Primary Mathematics Year 6 (Clarke, (ed.) 2000). In an everyday context, however, one might use and interpret 'even chance' in a more vague manner, for example sometimes it is used to replace the phrase 'equal chance' or the 'same chance'. My purpose is not to discuss whether the phrase 'even chance' is appropriate or acceptable in primary mathematics classrooms. It was the phrase presented in the mathematics textbook and used by the teacher, in a 'typical' primary classroom situation, and thus it was my focus. My purpose is to analyse dialogic practices used by the teacher and the children when they explain the meaning of the phrase 'even chance' and other mathematical words.

### **Research questions**

The investigation of contextual references used to describe meanings of specific mathematical words in a particular school setting is sociolinguistic in nature. Sociolinguistic research studies the relationship between the language and the situational context in which it is used (Holmes, 1992). The educational setting is a broad social setting, and the means by which knowledge becomes 'shareable' is language. I also consider social aspects such as the pairings of children, the opportunities they have to speak, or communicate by other means. I am taking a linguistic approach to studying the language with a focus on description, in relation to the function of conveying meaning (Hudson, 1988). My research is about constructing

theory more than testing theory and I am interested in understanding the nature of mathematical language as it occurs in classroom situations. Thus, the pragmatic aspects of language are of more interest than the semantic aspects. My aim is to analyse classroom discourse to discover the ways in which teachers and children share the meaning of specific mathematical words in different settings. Although spoken language is the focus, other communicative events are also an important part of the social aspect of communicating in different situations. My research is also 'interpretive' in the sense that I am a primary mathematics education specialist applying my knowledge of primary mathematics teaching and learning to the data I collect (Graue and Walsh, 1998). Graue and Walsh (1998) believe that interpretive research with few subjects, over a sustained period, is a very appropriate way of understanding children. My research lasts a short period, but is sustained in the sense that I watch a series of lessons, and I spend 30 minutes or more with each pair of children. I involve only a few subjects and provide detailed observations. I am particularly interested in exploring contextual references occurring in discourse during explanation and exemplification of mathematical words. The contextual references are, in my view, a pseudo-social context, to which participants refer, particularly if they refer to real-life events. In this sense, the contextual reference might also become part of the immediate social setting for the dialogue, e.g. a child drawing trains on a railway line. The project has two interrelating strands exploring both *teaching* and *learning*. The ultimate purpose is to inform and improve classroom practice.

I have two broad research questions:

**1) How does the teacher interact with the children to develop a shared understanding of the meaning of mathematical words?**

In particular, my intention is to explore the specific contextual references used by the teacher and the children. The aim is to discover how such references affect the process of developing a shared understanding of the meaning of mathematical words. This clearly relates to the idea of 'precision', as the teacher might define a word very narrowly and guide

children to a 'shared understanding of meaning', or alternatively might provide a range of opportunities for children to construct their own meanings. The analysis will consider the effects of various contextual references in either the teacher's or the children's contributions to dialogue during the lesson. It is important to consider the effectiveness of the dialogue between the teacher and the children, in relation to the children developing a shared understanding of the meaning of the mathematical words. The analysis will look closely at how the teacher introduces mathematical words and how dialogue involving mathematical words evolves with the whole class. I will also consider whether the children have the opportunity to use the words and to explain their meaning. In particular, I consider the effectiveness of using a specific contextual reference in relation to each member of the class having the same understanding of meaning of the word.

## **2) How do children express their understanding of the meaning of specific mathematical words and phrases?**

My focus will be on the contextual references used by the children in their explanations of mathematical words. One interest is whether the children refer to the same contexts as the teacher or whether they prefer to use others. Another aspect I will consider is whether the contextual reference is helpful in explaining the meaning of a word, or whether it creates difficulties for the speaker or the listener. It will be interesting to explore the ways that children support, or challenge, each other's explanations. My analysis will attempt to identify the level of dependence children have in setting a mathematical word into a particular context. As before, my focus is on the ways that children share their meaning of the word, and the means of expressing their thoughts i.e. *their dialogic exchanges with peers.*

### **Summary, and outline of subsequent chapters**

In this chapter, I have set the scene for the iterative process of research that will involve much revisiting and refocusing of ideas as I become more familiar with the available data. I have provided an overview of the nature of mathematics language, the need for it, and the possible confusion arising

from its use. Current linguistic research focuses on observing participant dialogue and the development of shared understanding of meaning. This project aims to focus on the communicative methods used by teachers and children when sharing their understanding of the meaning of mathematical words. References to recent legislative documents from government agencies have generated discussion of the notion of 'precision' in mathematical language, especially in relation to the use of mathematical words in 'everyday' contexts. This developed into consideration of the teacher or child making a 'contextual reference' in order to explain a mathematical word. At this stage 'context' refers to a *physical situation* or a *mental representation* of one, e.g. 'sharing pocket money'. Initially my focus is on the actual contextual references in use, by the teacher and the children. My purpose is to consider how such references help teachers and children develop a shared understanding of the meaning of mathematical words.

Below is the outline content of future chapters:

Chapter 2: Literature Review - Sets out to discuss issues in more depth, including the notion of context in relation to language.

Chapter 3: Research Methods - Provides a brief overview of the pilot study and its purpose in trying various methods of collecting classroom discourse leading to alterations in the two strands, lesson observations and structured interventions, in the final phase of field work.

Chapter 4: Analysis – Provides some brief analysis of the pilot research phase and its influence on subsequent analytical methods of the final evidence base. Analysis and identification of key points for each strand

Chapter 5: Conclusion – Summarises the key findings, discusses implications for teaching and learning, and my research contribution to theoretical understanding of teaching and learning. It also evaluates the whole project and considers further questions arising from the findings.

## **Chapter 2: Literature Review**

### **Introduction**

This review is in three broad sections that provide a sound literature base to support my analysis. Each section is distinct in nature and emphasis but together the sections provide an interrelated web of ideas. The first section presents contrasting views from work on the 1987 – 1991 National Oracy Project and other sources about the need for a specialist language in subject teaching. The work of the National Oracy Project (Norman, (ed.) 1992) is an important reference because it is classroom-based research, often conducted by teachers, that informs practice. Through such reading, I explore possible areas of difficulty in sharing the meaning of a specialist vocabulary. This section is highly relevant in the light of previous discussion in Chapter 1, of government initiatives that promote the teaching and learning of a precise mathematical language.

The second section develops a conceptual framework for the research with reference to a range of perspectives but particularly including ideas from social constructivism in mathematics education, psychology and linguistics. There are tensions in attempting to draw upon different fields of research, but each provides a different perspective that has relevance for the situation under study. There are also commonalities and identifiable parallels to explore and the crossing of such research boundaries is becoming more common in educational research. I consider Vygotsky's (1962, 1978, 1987) influence first, because almost all subsequent references pay heed to his work. I also refer to Bakhtin's (1981) work to illustrate the notion of dialogicality, which is important for the analysis. Various other more recent sources provide a background web of ideas from which to develop a clear analytical framework. Consideration of how understanding of meaning becomes shareable leads into consideration of the pedagogical issues in teaching children a mathematical language. As part of this discussion, I look at the work of Piaget and Inhelder (1975), and others on the development of probabilistic concepts and language.

As 'context' influences the 'potential to make meaning shareable' the third section addresses some of the complex and interrelated issues about what is meant by 'context' in this study. I have already introduced the idea of 'contextual references' in Chapter 1, and it is important to explore the various ways that others have interpreted 'context' and its possible influence on communication during the teaching and learning process. At the end of this section, I include a short discussion of the place of probability as a mathematical topic, and the type of contextual references used to exemplify understanding in much current British primary school practice. This whole section identifies the analytical strands to follow when focusing on the effectiveness of communication.

### **The need for a specialist language**

Each subject area of the curriculum has its own specialist terminology. Whether it is necessary for children to know, understand and use this terminology is the subject of debate in various educational research communities. Chapter 1 highlighted the recommendations of the Cockcroft Report on the teaching of mathematical language in England and Wales. The report implies that mathematical learning requires the development of a mathematics register although the report does not describe it as such. A mathematics register is the meanings that belong to the language of mathematics, including natural language and not just consisting of mathematical words and phrases (Halliday 1978; Pimm 1987; Griffiths and Clyne, 1994). A mathematics register is useful in specific situations that require it e.g. mathematics lessons. It helps to define the subject boundary, and the nature of being a mathematician (Brilliant-Mills, 1994). Teachers might view children's communication of mathematical ideas in their own language as less accurate than presenting the same ideas using specialist terminology. Such teachers might have the view that use of a specialist language signals membership of a particular group as a source of status and identity and that it signifies that mathematical learning has taken place. In this situation, specialist language forms an important socio-cultural function for those who use it and they think it necessary for the development of 'disciplined' thinking i.e. the ability to think about a specialist subject

discipline such as mathematics. This seems to be the view promoted by the National Numeracy Strategy and Ofsted as described in Chapter 1.

One might view a subject discipline as a language game, i.e. it has characteristic logical structures, particular concepts, and distinctive kinds of test for the truth of propositions (Hirst and Peters 1970, cited in Barnes and Sheeran, 1992). A child has to learn the rules of the game in order to participate in mathematical dialogue, and thus learn mathematical concepts. Links between the ability to communicate using specialist language and learning the subject matter are unclear (Pirie and Schwarzenberger, 1988; Bennett, 1996; Pimm 1997). Communication in non-mathematical language also forms part of the mathematical register, as described previously. It might be that the ability to communicate mathematics effectively does not necessarily require the specialist vocabulary. Pimm (1997) described pairs of Canadian students from a range of cultural backgrounds, discussing their computer-based mathematical activity in vague non-mathematical language, and suggested that mathematical language was not essential for effective communication of mathematics in this situation. They were able to point to the screen and say, “turn it this way”, “ move it up here”. Their mathematical understanding of the problem they were solving was evident in their communications and actions, but not explicitly stated in mathematical words. Bennett (1996) however, does consider technical vocabulary useful to communicate subject matter effectively during primary technology lessons. The social context must therefore determine the level of need for using a specialist vocabulary. Working in pairs at the computer allows the use of gesture, vagueness, presentation and testing of tentative ideas, but when working in whole class situations a specialist vocabulary might improve effectiveness, assuming everyone knows the language. In my own experience, primary school children in general are not used to working with a teacher who presents tentative ideas for debate, but are quite happy to learn and use, particular mathematical language in a structured way. In smaller groups and especially with computer problem-solving children seem able to present tentative ideas because of the nature of the activity. It seems that in the whole class situation the children have an expectation that the



teacher will tell them what to do, and that there is a 'confidence zone' that prevents them from pursuing tentative ideas in their own language for fear of failure. Before the introduction of the National Numeracy Strategy in many British schools during September 1999, many teachers did not expect, nor encourage children to develop a technical discourse in mathematics but focused on teaching mathematical procedures (Rowland, 1995). Because children seem to learn a lot of mathematics without developing the ability to communicate using a specialist vocabulary, primary teachers might question the need for it. Those working with younger children or older children with under-developed language skills are particularly likely to question the use of specialist language in each subject. In contrast to this, earlier research undertaken in infant schools just after the Cockcroft Report, found that infant teachers considered that developing the children's mathematical vocabulary was their most important function (Clemson and Clemson, 1994). However, much of the mathematical vocabulary development in the infants at that time was using language that is also used every day i.e. positional language and comparative language. This is still important today, but there is now much more emphasis on children at Key Stage 1 using and understanding words describing operations on numbers or properties of shapes 'division' or 'symmetrical'. The main drive for this is the requirements of national testing at Key Stage 1, which influence the curriculum content and set national expectations.

Linguistic research, particularly in Australia, has shown that successful learning in any school subject requires abandonment of 'taken for granted', 'everyday' and 'common sense' meanings for more precise specialist meanings (Barnes and Sheeran, 1992). Such research seems to support the view that knowledge of a specialist mathematical language is essential. Australian linguists argue that an emphasis on everyday language denies pupils access to important discourse modes or genres, essential language forms for carrying the meanings of specialist subjects (Christie 1985, cited by Barnes and Sheeran 1992). Teachers are usually the 'language-definers' in the classroom and many view successful learning in specialist subjects, as one of taking on the teacher's definitions. The contrast between the 'school'

meaning and the children's pre-existing or 'alternative' meaning is often lies at the root of misunderstandings. Clearly, such research findings reinforce the view that using everyday language may lock the children into everyday ways of thinking. Support for learning a specialist subject language appears strong within the linguistic research community. My own research does not aim to support or refute the idea of needing a specialist language in mathematics lessons, but aims to identify and analyse the communicative processes when attempting to share the meanings of such language during classroom dialogue.

During my final research phase, the teacher chose to teach three consecutive lessons on the mathematical subject 'probability'. One of the difficulties with probabilistic language introduced at primary school level is that much of it also has 'everyday' meanings. Unfortunately, most research into children's learning of probability focuses on conceptual rather than linguistic development, but researchers often use language to determine children's understanding. During the late 1940's Piaget and Inhelder researched children's understanding of probabilistic concepts. Their research included a significant use of reporting and analysing children's dialogue to explain their ideas (Piaget and Inhelder, 1975 translation). However, the emphasis was on identifying levels of conceptual understanding of probability rather than the development of a probabilistic vocabulary. Children explained their ideas, and answered questions during experimental situations, using their own language. The language of chance was unexplored because Piaget and Inhelder were not researching how language helped children learn or explain probabilistic concepts. Nor were they focusing on the development of a mathematical register with which to explain ideas. They made distinctions between the qualities of children's answers by making judgements about precision in children's explanations. Further discussion of this approach to research occurs in the next section of this chapter when developing a conceptual framework for my own research. Recent research into children's ability to make probabilistic judgements has shown there is a linguistic factor that affects the sense a child makes of the mathematics involved (Fischbein, Nello and Marino, 1991). The research

team discovered that a significant number of children in Italy, ages 9-13, had greater difficulty with determining whether some events were 'certain' than whether events were 'possible'. For example, some children considered the probability of rolling a number less than seven on a die as 'possible' rather than certain, because they viewed each number on the die separately as having a 'possibility'. A significant number of children also had difficulty in separating events that were 'highly frequent' from 'certain'. The team concluded that not all children had a clear definition of the word 'certain'. It seems therefore that children developing unclear meanings of probabilistic language might affect the development of mathematical meaning. My analysis aims to identify some of the reasons why children acquire unclear meanings of such words through classroom dialogue.

A word acquires properties or meanings by association with an artefact or event (Wallwork, 1985). Thus, children give meaning to words in the immediate context at a particular time. In giving meaning to the words, they also accommodate new information with old. Some specialist words are therefore easier to learn than others are because they only occur in the specialist context. A striking example of this is that Welsh children responded more accurately than British children did to the use of 'similar' in geometry, because no word exists for 'similar' in everyday Welsh (Rowland, 1995). The mathematical register includes specialist words and particular grammatical forms used to express and explain mathematics. White, (1988) in Barnes and Sheeran, (1992) wrote that some educators believe that the use of particular linguistic forms directs pupils to differing ways of thinking and understanding. An example of this can be taken from the recent National Numeracy Strategy training pack suggesting that children say 'forty minus twenty' instead of 'four minus two' when subtracting tens (DfEE, 1999d). The purpose is to encourage the children to think in 'tens' rather than single digits. In practice, when associated with physical models, and expanded-written models of calculations, it appears to help children's mathematical development. White (1988) also emphasised the importance of children being able to use a subject specialist word or phrase that encapsulates a set of complex ideas. For example, in

mathematics the name of a particular shape signifies to the communicants its properties. Problems arise when the set of ideas a word or phrase encapsulates, does not match the set of ideas held by another participant in the dialogue. For example, the teacher connects the word square with the following properties: four right angles, four equal straight sides with opposite sides parallel, and belonging to the set of rectangles, parallelograms, rhombuses and quadrilaterals. For a child the ideas the word 'square' evokes will depend on age, experience and stage of mathematical development. A very young child will say it is a box, an older child will describe it as a shape with four equal sides and four corners. Only over a period of years will the child come to comprehend the multiplicity of meanings of the word 'square'.

A mathematical register or discourse mode is for making and sharing meaning. Meaning also comes from the situation in which communication takes place. When we communicate we have our own concerns, but we are also mindful of the concerns, needs and interests of the listener. Generally, deliberate choice of words does not direct our speech during conversation or discussion. In these situations, we are more aware of the purposes of the talk than of the language forms we use. However, a teacher often chooses specific words or phrases during exposition e.g. even chance, in order to introduce them to the children. Later in the lesson, according to Christie (1985) in Barnes and Sheeran (1992), an essential element in learning the new word or phrase is group discussion. Discussion provides children with an opportunity to negotiate linguistic structures as a way of thinking about the subject. This supports the 1982 recommendations of the Cockcroft Report and recent recommendations for changes in British teaching methods include 'direct interactive teaching' strategies with the whole class (DfEE, 1999a). During 'direct interactive teaching' teachers ought to use a variety of communicative strategies including explaining ideas, modelling procedures, questioning, listening to, sharing, developing and challenging pupils' ideas and ensuring all children are involved through the use of appropriate resources. It is essentially a multi-sensory approach aiming to develop connections between mathematical ideas, and to teach children how

to explain their methods, provide reasons and justify their answers. Teachers in Japanese mathematics classrooms sampled by the Third International Mathematics and Science Survey [TIMSS] (National Center for Education Statistics [NCES], 1998), commonly used this approach and the researchers judged it very effective. Recent British research by Askew, Brown, Rhodes, Johnson and William (1997), supported the development of a rich connection of ideas in mathematics. Such development cannot occur without engaging in discussion to develop shared meaning. Askew *et al.* (1997) found that effective teachers of mathematics used pupils' reasoning and oral descriptions of their mathematical methods to help them establish connections between ideas. Such teachers believed in challenging children to explain their ideas, an approach also promoted by the National Numeracy Strategy Framework (DfEE, 1999a). It is clear that teachers can model the behaviour and language associated with the subjects they teach, but they also need to engage in discussion to develop a shared understanding (Edwards and Mercer, 1987). Edwards and Mercer demonstrated that teachers often communicate interactively the appropriate patterns of working and thinking to their pupils. The children learn the language and associated behaviour through modelling over a period. A skilful teacher carefully builds conceptual structures whilst explicitly associating them with new mathematical words and phrases. During the National Oracy Project teachers found that developing children's ability to use specialist terminology alongside the acquisition of new concepts was a lengthy and complex process (Norman, K. 1992). They supported Edwards and Mercer's (1987) assertion that children need to explore and process new words and phrases in their discussions. In particular, peer discussions promote confidence in using the words again in teacher-pupil dialogue (Johnson, Hutton and Yard, 1992). Although I agree that peer discussions are a valuable teaching and learning approach, in my experience they do not necessarily lead to children using specialist terminology unless such use is a specific objective of the discussion.

Teachers and children can also discuss the subject without using specialist vocabulary, and in some cases, shared understanding of meaning occurs

more quickly if specialist words are abandoned. Hence, when introducing new ideas to young children teachers will use everyday words to describe 'mathematical' concepts. When young children classify shapes by their ability to *roll* or *slide*, they are using words from their everyday experience. *Roll* and *slide* are non-mathematical words that describe the physical phenomena resulting from mathematical features. The children then learn to classify using the mathematical features 'curved' and 'flat', thus gradually 'mathematising' the vocabulary. By the end of primary school, a teacher expects more specialised reference to angle, symmetry, and regularity when sorting shapes. This type of progressive development does not necessarily occur with every mathematical topic, and in particular, development of probabilistic language might not follow a progressive pattern. The lack of a specialist vocabulary might exclude a child from particular groups, by affecting performance in tests or interviews. The report on the Key Stage 2 national curriculum assessments from the Qualifications and Curriculum Authority (QCA, 2001) comments that children were poor at answering oral questions that tested knowledge of vocabulary. There seems to be a strong argument for children learning a specialist vocabulary although much current social constructivist research in the field of mathematics education that I discuss in the next section, seems to focus on children learning mathematics through their natural language.

### **The role of language in learning**

Many educational writers refer to Vygotsky (1962) *Thought and Language*, when considering the relationship between language and learning, although Vygotsky (1962) did not specifically focus on education, but was researching within a psychological framework. To provide a theoretical background for my own research I examine the origins of current thinking about the importance of language in developing shared meaning. One particular focus of my research is the ways children and teachers share their understanding of meanings of mathematical words using contextual references in their descriptions.

It is appropriate at this point to broaden the concept of a contextual

reference in relation to classroom practice. My definition from Chapter 1 is, “A ‘contextual reference’ is a spoken or recorded reference to an actual *physical situation* or a *mental representation* of such, in which the child or teacher perceives application of the mathematical word.” Some examples of physical situations are mathematical equipment; a role-play area; a textbook illustration; a drawing or a gesture. These are all *physical* in the sense that they are visible, and/or tactile. They can become *mental representations* of situations in the recipient as the recipient tries to match their perception of the situation with previous knowledge. A *mental representation* is also a situation held in the speaker’s imagination, derived from memory, and described in relation to the mathematical word or phrase under study. The textbook used by the teacher set up a *physical situation* through words and pictures. It expected the children to *imagine* they were in Atholl Wood, and to relate the questions asked to this contextual reference. This is the typical approach of many school texts. In relation to explaining the meaning of a specific mathematical word or phrase, my hypothesis is that contextual references most likely arise from the speaker’s ‘imagined’ situational context that they perceive the word to describe. From this imagined situation, the mathematical word links with other words to create a phrase, or a sentence that helps the recipient develop meaning for it. The speaker might accompany utterances with apparatus, drawings or gestures. A specific example from probabilistic language is the word ‘impossible’. A child might describe ‘impossible’ as, “It would be impossible for me to squeeze through a small hole (gesture to show size)”. The recipient might personalise the contextual reference to hole and think about his or her own experiences of squeezing through a small space. Thus, whether the recipient agrees with the explanation, and derives the same understanding of meaning from it depends on the recipient’s own experiences. I will continue this discussion later in this chapter when I consider the variety of contextual influences on the development of meaning, and in Chapter 4 when I use the ideas in my analysis.

The ‘meaning making’ process, occurring in thought, transfers into speech. We might describe ‘meaning’ as a union of word and thought, which is a

dynamic rather than a static formation (Vygotsky, 1987). Vygotsky described thoughts expressed in words as 'inner speech'. Inner speech shows a tendency towards predication, because speech for oneself can make sense in a different form than external speech. Inner speech seems to be an essential part of the process of developing external speech. The notion of 'inner speech' being a mental draft or mediator between thought and word seems to fit with my hypothesis of how contextual references come to be used to explain word meanings. Vygotsky (1987) claimed that words only make sense in context, and that a word changes its sense in different contexts while the meaning remains the same. The context is either the external situation or the internal psychological context. A word also derives its meaning from its position in a sentence, and intonation in speech can alter the meaning of a sentence. Within the classroom situation the child often has visual and aural information presented together. If the language does not match the visual image then perhaps the child develops a different meaning of the event than that intended by the teacher. Vygotsky (1962) claimed that to understand another's speech we must also understand his or her thoughts, and the motivation for the speech. A teacher who shares learning objectives with the children might be moving some way towards providing the motivational reason, but if the teacher's words and actions cannot help the children access his or her thoughts then opportunity to share meaning is lost. Alternatively when children engage in peer talk on a common task they have a shared motivational reason in it. Thus shared tasks involving discussion between peers ought to help children develop a shared understanding of meaning. When a child knows the meaning of a word the child can then use the word to promote and develop his/her own actions and ideas. This process is developmental and as the vocabulary grows so does the child's capacity to develop logic and abstract thought. Although this seems to be a clear development, in reality the learning of word meanings and the different sense they make in different contexts is most likely a complex weaving together of different layers of thought and language.

In *Mind and Society* (1978), Vygotsky (1994) considered ways that children might learn through interaction with adults. His assertion was that adults and



children agree on a reference e.g. both use the word 'dog', but may fail to agree on meaning e.g. the child might think 'dog' refers to one particular dog, while the adult has a generalised view of 'dog'. This mismatch provides the main impetus for development. The "zone of proximal development" described by Vygotsky is the distance between the child's actual developmental level in independent problem solving, and the potential development through problem solving with a more capable person (Vygotsky 1994, Wertsch, 1985). In other words, the teacher works with the child at a higher level than the child can achieve independently. Dialogue is an essential element of the process. Vygotsky's theory rests upon the assumption that the child makes a preliminary hypothesis about the meaning of a word, and through interaction with adults, the child refines it. Vygotsky (1962) took the view that knowledge of language structure is not a prerequisite for making and sharing meaning. He clearly viewed the ability to generalise word meaning as a social process, in which dialogue plays an essential role. 'Generalisation' is also a mathematical communication skill in developing ideas of 'proof'. We eventually express generalisations and proofs in mathematics in abstract algebraic forms. Language thus creates the possibility of developing abstract thinking.

Vygotsky wrote about children first using words as referents and later making sense of them (Lee, 1985). In mathematics, a possible example is a young child recognising the number symbol 4, and naming it 'four', but not linking 'four' to a set of four objects. Vygotsky would say that the child has a "spontaneous concept" of four. Once the child links the symbol to a set of four objects, the child makes sense of it and has a "scientific concept" of four. Vygotsky called this upward development of scientific concepts and considered this to accompany a decontextualisation, suggesting that the child can use abstract thoughts alone to mediate action. Decontextualisation is speech with linguistic units that relate to each other, but are independent from their everyday reality (Wertsch, 1991). In contrast, Vygotsky also described 'inner speech' as something that serves to create its own context or "recontextualisation". He considered that 'sense' predominated over 'meaning' in inner speech. 'Meaning' is a fixed entity

linked to the semantic aspects of language while a word changes its 'sense' in various contexts (Wertsch, 1991). An example of a decontextualised meaning might be that the word 'four' always signifies a count or set of four. A child 'recontextualising' might interpret the number 4 in 44 as 'four and four' because the number forty-four is unfamiliar. The relationships between decontextualised and recontextualised meanings are very complex. The speaker is unaware of the recontextualised meaning that the recipient is attaching to an utterance. As soon as language describes a mathematical situation, then the compatibility of what the speaker says with what the recipient hears has to rest on shared understanding of meaning.

While Vygotsky focused on using word meaning as a unit of analysis, Bakhtin considered the whole utterance as the most meaningful unit of communication (Wertsch, 1991). Although Bakhtin (1981) was writing about discourse in the novel, his work refers to all types of discourse and highlights important issues to consider about the nature of discourse. He believed that an utterance is unique and wholly bound up in the situation. Each utterance in a conversation builds upon the previous one, depending on the sense made of it by each participant. This is dialogicality, or multiple authorship of a spoken text. For the purposes of exploring classroom discourse, Bakhtin's view of the whole utterance is important, in relation to the ways children share meanings of mathematical words or phrases. Bakhtin (1981) also considered each utterance to have a history and once spoken it becomes exposed to contradictions caused by the context. A word has meaning according to its dialogic orientation among other words i.e. it becomes individualised and stylised according to the immediate environment. The speaker gives intention to the spoken word i.e. its intended meaning, but a range of influences affect the whole utterance to make it an active participant in dialogue. We might think of the teacher in the classroom saying, "If I toss a coin there is an even chance of *heads* or *tails*". What this means to each child depends upon the historical and immediate settings for the expression, and the knowledge and experiences that each child associates with the utterance in the particular situation at the time. The teacher has one intended meaning, but the children might develop

different interpretations. Bakhtin (1981) also wrote about a word having its own dialogism, meaning that it already has within the object it describes something that challenges its' meaning. In the act of saying a word, it provokes and anticipates a response. In this way, we understand meaning in relation to other utterances and not by one utterance alone. The immediate context or placement of the word in an utterance, or an utterance in a conversation, is therefore important. Bakhtin's ideas support mine in two important respects. The first is that the meaning of a mathematical word or phrase is part of the whole utterance, which in my research might include a contextual reference. Thus, the contextual reference is bound to affect meaning if Bakhtin is correct. The second is that of intended meaning, because in both strands of my research I am looking at how participants respond to each other's 'intended meaning'.

Children notice definitions of words through speaking, using the words, asking questions and solving problems (Cobb, Wood, Yackel, and Mc Neal, 1992). There is a place for children learning how to use the words in practice. They must know *how* to use the word, to establish its meaning through a mathematical dialogue. In a mathematics lesson, we might assume that children, who cannot use mathematical words, have not understood their meaning. Two perspectives on the problem exist. Either the child is unable to **use** the word as part of a sentence in order to explain a mathematical happening, or is unable to **associate** its meaning with a mathematical concept or concepts (Pirie and Schwarzenberger, 1988). I am not concerned with determining levels of mathematical understanding, but I am interested in communication processes involved in the children and teacher sharing their understanding of the meaning of mathematical words. I expect them to use the word or phrase as part of a sentence, or other communicative event, in order to explain the associated mathematical meaning. The actual process of sharing meaning is identifiable through the dialogue and any other communicative events such as drawings or gesture. Children's cognitive work takes place in a social dimension before it takes place internally (Hicks, 1996). Hicks draws on Vygotsky's ideas and states that the child participates in a socially constructed activity and takes part in

discourse that enables thoughts to become internalised and under the child's control. Hicks (1996) also referred to Bakhtin's ideas about dialogic speech being at the core of thinking. Thus in order to explore the ways in which children share the meaning of particular mathematical words it seems logical to focus on the dialogues they engage in.

It is important for children to learn about *educational discourse* and how it operates, in order to learn and become members of a community that uses *educated discourse* (Mercer, 1995). A simple explanation of educational discourse is that there are expected ways of communicating within the classroom situation. The teacher is usually in control of both the physical situation and the dialogue. However, ability to take part in educational discourse is *not the most important educational goal*, but is the means that allows children to participate in teaching and learning. The most important goal is that children learn how to use educated discourse, which most likely occurs when the teacher and children or peer groups engage in discussion and debate about the subject of the lesson. The aim is for children to learn different ways of using language to think and learn. Teachers may model the modes of discourse for a particular subject, or might encourage children to construct such discourse practices in supported group discussions. As discussed in Chapter 1 a subject such as mathematics has distinctive conventions of thought and has particular conceptions of phenomena, and the teacher has to communicate these as useful ways of understanding the world. This process of constructing and transforming meaning requires time (Barnes and Sheeran, 1992). Mason (1999) wrote about mathematical words used as labels depending on their use and interpretation in a particular situation. Mason suggested that words such as *angle* support precision in thought and in justification and argument in a mathematical setting. Labels also trigger thoughts from the past, and enable associations with previous experiences. Within a constructivist philosophy, we might consider where the use of 'labels' fits, and whether reconstruction of ideas requires the development of new labels. For example, a child might be able to use the word *round* to describe a circle, and label the shape '*a round*'. When the child learns the name *circle* and uses it as part of the description, we might

say that the child has reconstructed the idea of circle.

Kieren's (1993) model of the recursive theory of mathematical understanding includes language at the level of '*image making*'. He described '*image making*' as the second level of effective action during which the child may work on some problems e.g. tossing a coin a hundred times and recording the results. '*Image having*' follows image making in which a mental object develops e.g. the child has an image of a coin with two faces and therefore equally likely chance of heads or tails. Developing the mental image requires recursion to the '*image making*' activities and the perception of a pattern within them usually expressed linguistically. Vygotsky's ideas have influenced Kieren's analysis in the sense of language helping the child form a mental image, as the mediator (Vygotsky, 1987). It is interesting to note that as Kieren (1993) described his model he provided examples of a child '*thinking aloud*' or expressing understanding in words. Although there were no further explicit references to language in this model of concept development, it seems obvious that the development of thought process through the model relies heavily on language to describe observations and make connections with previous understanding. Unfortunately, Kieren did not analyse language used as part of the research into the development of mathematical understanding, although it seems an important aspect of such development.

As discussed earlier, British primary school teachers are encouraged to employ '*direct interactive teaching*' methods with appropriate mathematical language to aid mathematical learning at different stages of schooling. Thus children begin with various practical activities to help them visualise a the meaning of a word or phrase e.g. picking coloured counters from a bag and making predictions such as '*more likely*' or '*less likely*'. They then move on to placing such predictions onto a probability scale and building firmer relationships between the probabilistic words or phrases, before working with quantification of probability in an abstract way. Thus, British education generally seems to follow Kieren's (1993) theoretical model, giving importance to developing imagery though language to ensure concept

formation. However, children have to accept responsibility for learning, and have a desire to develop a shared understanding of meaning through enquiry-based teaching methods. There is evidence from classrooms in Japan that children who fully participate in enquiry-based learning through sharing their mathematical ideas with the class, develop understanding of the language of the mathematics classroom (NCES, 1998). Observations such as these, from other countries, have led to the current national initiatives to improve mathematics teaching in primary and secondary schools.

Much recent research in mathematics education derives from a social constructivist philosophy. Social constructivism is a philosophy that when applied to mathematics is able to account for the nature, application and learning of mathematics (Ernest, 1991). Using such a philosophy, we can view the basis of mathematical knowledge as linguistic knowledge, especially language as part of social construction. Ernest describes constructivism as reconstructing mathematical knowledge in order to safeguard it from loss of meaning. More recently, Ernest (1997) has become interested in *semiotics, the social production of meaning, as a framework for describing mathematical activity and learning*. Vygotsky also presented what is essentially a semiotic view of learning (Lee, 1985). Semiotics draws together linguistic, cognitive, philosophical, historical, social, cultural and mathematical perspectives (Ernest, 1997). Both semiotics and social constructivism offer a theoretical basis for exploring mathematical learning that include language as a focus. Social constructivists tend to focus on the use of a child's natural language, rather than the sense that children make of specific mathematical words or phrases. They might use dialogue as a means of exploring the teaching and learning of mathematical understanding, not the nature of the actual communication itself e.g. Steffe and Tzur, (1994); Saenz-Ludlow, (1995). Ernest (1994) was critical of the social constructivist assumption that natural language is wholly adequate for presenting mathematical justifications and truths. However, the constructivist approach fits in with Piaget's theory that the child's understanding of the world is in relation to the context and path of their

development (Confrey, 1999). Confrey wrote about the child having understanding of the world that is complete, coherent and explanatory, for that child. Thus, mathematical understanding is personal, provisional and described using language that is within the child's own vocabulary at that time. According to Confrey (1999), the teacher's role is to listen to the child and provide new situations that cause the child to make sense of the situation in relation to their provisional understanding through language. However, my research focus is not on the mathematical understanding, but on the communication processes the teacher and children use in order to make meaning shareable. I focus on the construction **processes** rather than the **results** of constructing new meaning through language.

As mentioned before research into children's understanding of probability tends to focus on conceptual rather than linguistic development. Experimental situations demonstrated a progressive development in the understanding of probabilistic concepts (Piaget and Inhelder, 1975 translation). Piaget and Inhelder used the children's oral responses to identify their stage of conceptual development. They made a judgement about the quality of each response in relation to the question asked. When looking at children's predictions about centred distributions, they identified stage II as "*Beginnings of structuring a distribution of the whole and generalisation from one experiment to the next*". In their analysis of stage II subjects, they identified children that showed implicit understanding through their predictive statements, e.g.

"balls will pretty much go all over, but with more in the middle"

...and a child who was more explicit e.g.

"there are some balls that collide and after this collision there will be one that goes there and the other there; I saw some which touched as they were rolling. That makes them go to the side"

(Piaget and Inhelder, 1975 translation, page 44)

Piaget and Inhelder (1975) therefore used language as a means of making judgements about the developmental stage at which the child operated. They did not expect the children to use mathematical words, but to explain their thinking in their own words. In the examples given above the second prediction describes the possible causes as well as the result, while the first prediction describes the result only. If a child considers the causes in the prediction then this is a clear indication of Piaget and Inhelder's stage II. My main concern about such interpretation is that the researchers might assume that a child not speaking about the causes has not considered them. On the other hand, teachers have to interpret children's responses on a daily basis in their classroom teaching in order to inform their assessment. Interpretation in research has to occur at some point, whether it is the interpretation of statistics or a conversation. One must assume that the researchers have the knowledge and experience to make such interpretive judgements. Piaget and Inhelder (1975) were not concerned with dialogue, or the children's responses from a linguistic point of view. They were not trying to identify how children constructed meaning through language, but showed interest in the nature of their predictions, and how they constructed mathematical understanding through observing experimentation after having made a prediction. The work completed by Piaget and Inhelder (1975) in the 1940's is different from mine in three respects. First, they used structured experimental situations to explore specific probabilistic concepts, while I am using 'everyday' sayings that include contextual references. Secondly, as discussed previously they used language to make judgements about conceptual development, while I am researching how children share their understanding of meanings. Finally, they make no comment about specific vocabulary, while I am exploring the meanings that children give to specific mathematical words and phrases. However, I can identify from their research the 'stage' of probabilistic thinking of a ten-year-old child, and some of the explanations they used. The nature of these explanations might be useful for comparison in my own analysis even though the purpose of the research is very different.

Other recent probabilistic research also focuses mainly on the conceptual



developments rather than the linguistic nature of probabilistic judgements. The introduction of the National Curriculum for Mathematics (Department of Education and Science, 1989) meant that probability was included in the primary school curriculum. The outline progression was to develop ideas of chance in Key Stage 1, through to further development of probabilistic language in lower Key Stage 2, and understanding a numerical probability scale in upper Key Stage 2. Much of this was a pre-formal stage of learning probability, while at secondary school children moved on to the formal stage involving fractional quantities and calculations. For many primary teachers this was a new topic to teach, and was often one they had studied in little depth themselves at school. The links with handling data were unclear in the documentation, and examples provided led people to believe that introduction of the language with some activities involving chance, was all that was necessary. As a result, children had limited experiences and still entered secondary school with little conceptual understanding although they were generally familiar with probabilistic language. Children need many practical experiences of chance to put them in a better position for a more formal approach at secondary school (Straker, 1988).

Steinbring's (1991) analysis of a lesson on probability explores the relationship between language and conceptual development and illustrates the contradictory nature of a methodically organised presentation during a mathematics lesson when compared to the complex circular nature of stochastic knowledge i.e. chance. He suggests that the structured and organised methods of teaching probability negate understanding the nature of the subject. Teachers and texts give words such as 'chance' static definitions when they are dynamic concepts. Steinbring's (1991) analysis of a 5<sup>th</sup> Grade lesson dialogue shows chance being illustrated to pupils as a justification for an unexpected event i.e. when no physical law can explain the event it must be 'chance'. Thus, the teacher and children link 'chance' with improbable events only, when it actually has a much broader conceptual application. What he noted from the transcript was that the teacher tended to narrow down the options so that children became limited in their ideas, rather than opening out the options to allow them to explore a

range of avenues. He comes to a similar conclusion as Straker (1988) that elementary instruction in probability does not prepare pupils for further conceptual development.

The concern with programmes for teaching probability appears in other more recent work and Jones, Langrall, Thornton and Mogill (1997) developed a framework for assessing and developing children's probabilistic thinking. Of the four key probabilistic constructs in their framework, 'probability of an event' is most relevant to my own research. They identified different levels of thinking about events from subjective through to numerical reasoning. In relation to language, they looked at justifications expected from children at different levels. Their original framework made statements to indicate that a child at level three distinguishes between 'certain', 'impossible' and 'possible' events with a numerical justification. A child at level 1 will also distinguish between these types of events but in a more limited way. Thus, they expected children at different levels to know and use the same words, but with different levels of meaning for them. Jones *et al.* (1997) focused on conceptual development, using experimental situations for events with numerically expressed outcomes. The questions asked included probabilistic language and responses were analysed according to their framework. They revised their original framework and at level 1, it now stated that children recognise impossible and certain events. They based this change on one test that involved a gumball machine with only one colour of gum in it. Recognising 'certain' and 'impossible' was considered by Jones *et al.* (1997) to be the starting point for probabilistic thinking. Previous research questioned the ease at which children identified probabilities of certitude (Fischbein *et al.* 1991). Fischbein *et al.* found that children found it easier to identify events that were possible, rather than certain. They commented on the nature of events affecting responses, and the nature and language of questions asked about events and their influence on children's responses. One of the questions was, "By rolling a die, does one obtain a number smaller than seven?" and 37% junior high school children in the sample got it wrong. The context in which the question is set might have had an influence. This question certainly seems more

challenging than identifying the chance of picking a green sweet from a jar of four green sweets that Jones *et al.* (1997) used. It seems from such research that although the researchers were not focusing on language or dialogic processes per se, there is clear indication that linguistic factors are important in the development of probabilistic concepts. Each of these pieces of probabilistic research attempts to determine the challenges facing children when learning probabilistic concepts beyond the subjective intuitive stages of thinking. They analyse children's responses and reasoning then make judgements, based on the researchers own experience and subject knowledge about children's levels of thinking. My own research actually focuses on the subjective and intuitive, because I am trying to determine the effects of particular contextual references have, on the communicative processes of sharing meaning.

Mathematics education researchers, and others, often try to combine different theoretical fields in order to discuss classroom practice. For example, Tony Brown (1997) draws on hermeneutics, the theory and practice of interpretation; critical social theory and linguistics to inform his exploration of the way in which language and interpretation underpin the teaching of mathematics. His work provides some interesting insights into the relationship between language and mathematics. Brown (1997) stresses the individuality of our descriptions of the world, and the influence of culture. In constructing new mathematical ideas, children always make use of partly constructed language from their culture. Classroom discourse is always framed by the teacher who contextualises and conditions any terminology used (Brown, 1997). He also supports the idea that language is not just for description, but is actually part of the performance of mathematics and children have to learn the conventions as well as having the opportunity to construct their own meanings. Not all kinds of talking contribute equally to developing such meaning. Barnes, (1992) describes two main functions of classroom talk as 'presentational' and 'exploratory'. Presentational talk occurs when the speaker is well aware of the needs of the audience and more often focuses on the expectations of the audience than on the speaker's ideas. It often occurs in response to teachers' questions. In

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contrast, exploratory talk is often hesitant and uses ideas and language already known in order to see what is possible. It will only happen when children feel at ease. Teachers need to ensure a balance of opportunities for both types of talk. Exploratory talk allows reflection during and after the activity, which is essential for critical thinking and modification of what we believe.

In an attempt to encourage talk, the majority of primary teachers over the last 20 years have organised pupils' seating in groups. However, research by Wood, (1985) and Bennett, (1984) reported that most group-work talk was not task-related (Des-Fountain and Howe, 1992). There is also a relationship between interaction in the group and the pupils understanding of the task. Des-Fountain and Howe (1992) assert that the prevailing view of many teachers is that talk amongst pupils helps them get the job done, but does not have a significant effect on pupils' knowledge or understanding. In their transcript evidence, there are clear indicators that through discussion the children are working on their understanding of the new information in the light of information they already have. Wood (1994) also supported the idea that collaborative work with peers often leads to better understanding of the meanings of mathematical words. The development of new and shared meanings takes time. Des-Fountain and Howe (1992) identified such conversations between children as 'dialogic' in nature, as described in my previous discussion of Bakhtin's (1981) work. This means that participants build on each other's ideas, support and extend, finish off utterances, shift the meaning and effectively construct a joint utterance and a joint meaning. Des-Fountain and Howe (1992) also highlighted that children viewed each other's questions as genuine, in comparison to when the teacher asked questions. This is because the children knew that the questioner did not know the answer, while a teacher usually does. The children in each of their examples, clearly valued the experience of being 'thoughtful'.

Teachers found from their own research as part of the National Oracy Project, that they usually talk too much, don't listen carefully to pupils and are poor at getting pupils to contribute their own thinking (Norman, 1992).

Primary and nursery teachers expressed concern that child-centred practice was not sufficiently differentiated or challenging. The nature and quality of teacher interventions did not realise the full learning potential of exploratory situations. Secondary teachers questioned the transmission model that they saw as too didactic and unresponsive to pupil needs. It is interesting to look at current educational practice in the literacy hour and the mathematics lesson, and to consider the balance of activities. The aim of both models is to provide all children with a variety of teaching and learning situations, but the level of truly exploratory work is minimal. The teacher generally guides any independent work. Both of these teaching and learning situations provide opportunity for development of oral skills in explanation. However, these are most likely the 'presentational' type of talk, than the 'exploratory' talk, previously described by Barnes (1992). Successful teachers have a wide repertoire of ways of dealing with children (Corden, 1992). Corden described earlier work by Wells (1988) identifying different teaching styles ranging from the 'teacher as expert' at one end of the continuum and the 'pupil as expert' at the other. The teacher should respond contingently to children's needs by respecting the knowledge and experiences that children bring to the situation. The ability to respond in this way is central in determining the success of the teacher in entering different kinds of interactional dialogue with pupils. Unfortunately, many teachers use one particular model the majority of the time.

Whichever model is preferred, questions seem to be a pervasive feature of teacher language (Wood, 1992). Wood cites research indicating that teachers across all ages ask most questions, and few pupil questions are noted. In everyday conversation question often follows question. Classroom questions are different from others as there is an expectation that the children will answer them. There is often an expectation they will be answered in particular ways, or using particular words. Classroom questions set the tone for a biased power relationship that is potentially threatening to children. Teachers test knowledge and ask children for reasons to determine their understanding. For Socrates, the aim of questioning was to engage in joint enquiry in the search for truth (Wood, 1992). Wood includes Dillon's,

(1982) observation that most teachers' ask 'closed' questions requiring short factual responses, which tend to inhibit intellectual activity. Training experiments to help teachers raise the cognitive demand of their questions did not work. Dillon reported that such questions did not promote pupils thought processes as intended. Findings in later research by Swift *et al.* (1988) showed that allowing a longer time of silence after posing a question improved the quality and extent of pupil's responses (Wood 1992). The result was longer and more thoughtful answers. For children to develop a mathematical register, thinking time for constructing answers is essential during whole-class interactive teaching.

Perhaps teachers should listen more often and more attentively to children, thus encouraging answers that are more thoughtful (Brierley *et al.* 1992). Another suggested strategy is to engage in longer exchanges with a child, by asking further questions to encourage depth of thought (William, 1999). William found that this rarely occurs and suggests it is because teachers have difficulty changing their questioning styles, partly because of children's expectations. One can also teach by asking questions (Kamii and Livingston, 1994). In her own classroom, Livingston avoids direct instruction except for providing social knowledge. However, the social knowledge described by Livingston (Kamii and Livingston, 1994), how to write "*twelve hundred*" is clearly mathematical in nature. It might also be that strategies such as pretending not to know answers in order to get pupils to think, e.g. "I wonder if..." type questions, are one way to improve children's responses. This type of utterance signals acceptance of speculation and alternative answers and is becoming more common in English primary mathematics classrooms. Livingston suggests developing a climate that provides a context for talk where mistakes can be made, individuals move at their own pace and everyone's ideas are valued. Pupils will feel free to engage in fruitful dialogue and take risks when they know the teacher appreciates spontaneity.

It is important to focus on teacher-child interactions in order to examine the effectiveness of learning processes (Mercer, 1992). Mercer believes that an

important function of talk is that of developing shared understanding. Many teachers use questions to assess children, but it is also possible to use them to draw out ideas and guide learning. Different ways of responding to wrong answers can help to change an assessment situation into a productive learning situation. For example, a teacher may use what Mercer (1992) calls 'cued-elicitations' in which the teacher gives heavy clues about the answer either by wording or intonation. A danger in this approach to questioning is that the teacher perceives the child as 'knowing' a particular piece of knowledge, when in fact the child cannot recall it unless the question is spoken in a particular way. Cued elicitation is similar to French and MacLure's (1981), pre-formulating discussed by Cazden, (1988). Pre-formulating occurs when the teacher prefaces the question with an utterance that orients the children to the relevant area of experience that helps them to answer the question. Imagine some children looking at a picture of a triangle. An example of pre-formulating by the teacher is, "Can you count the sides and corners and tell me the name of the shape?" A nuclear utterance (one that does not pre-formulate) would be, "What is it?" Re-formulation occurs when the initial answer is wrong. French and MacLure (1981) identified five types of re-formulation providing different degrees of making the original question more specific with the cognitive task progressively decreasing (Cazden, 1988). The critical difference between helping a child get an answer and helping a child gain conceptual understanding is seen when the child demonstrates ability to answer similar questions to the original. It seems logical to view the learning of particular terminology for mathematics in this way. The teacher might pre-formulate or re-formulate to help the child use a particular term, but whether a shared understanding of meaning develops is unclear. The teacher must be aware that such strategies are to support learning, so that future questions might not require them. 'Joint knowledge markers' are references to some previous learning that both children and teachers engaged in and 'reconstructive recaps' to highlight significant features of the lesson (Mercer, 1992). In effect, the teacher is attempting to reconstruct the learning situation to ensure the child learns the objectives. The teacher often presents knowledge in such a way that it is not questionable. Thus, children have little



opportunity to develop their own meanings, but instead engage in a dialogue with the teacher leading to a consensus that the teacher has already decided (Edwards, 1990, cited by Edwards and Westgate, 1994). The teacher often controls the dialogue by determining the topics, allocating turns, providing feedback or running commentary on what is being said, thus providing the cohesion within the lesson and between sequences of lessons. Edwards and Westgate (1994) provide an example in which a teacher gave a narrow definition of a specialist word. When a child did not initially understand the particular word the teacher narrowed the curriculum in order to be sure the child understood the word. There are tensions between the teacher following clear teaching and learning objectives, and the desirability of allowing children time to make sense of ideas in their own way. We must consider how effectively schools develop 'meaning relations' within mathematics (Walkerdine, 1990 reprint).

The effectiveness of whole-class interactions is worth exploring (Jarvis and Robinson, 1997). Such exploration is particularly relevant in primary classrooms where teachers use interactive teaching strategies promoted by the National Literacy and Numeracy Strategies. Although Jarvis and Robinson (1997) did not research mathematics classrooms, their discussion and findings are useful. They used Vygotsky's suggestion that 'verbal meaning' has an important role in helping children have conscious awareness and be in control of what is learned. They focused on the teacher's responsive use of children's answers, explored the functions responses served and the discourse patterns that developed. Attempting to categorise 'situated' utterances was difficult, but their observations helped them define a pattern of publicly articulated interaction by which meaning becomes potentially shareable. First the teacher will 'Focus' on the topic and make it public. The topic is then 'Built' through interaction with the pupils. There is then a 'Summary' of the point or principle for that part of the lesson. Publicly articulated meaning becomes potentially shareable, but cannot guarantee learning. The pattern is a variation of the IRF (Initiation-Response-Feedback) model re-evaluated by Wells (1993). He cites research by Nystrand and Gamoran (1991) that suggests different levels of student

engagement with the topic occur with different modes of feedback. Wells (1993) presented a framework in which the meaning potential of a situation comes from activity, and discourse. If feedback challenges and explores the child's interpretation of a situation then it leads to the construction of new meaning.

This section of the literature review has drawn on a range of theoretical backgrounds in order to identify the issues that affect the teaching and learning of mathematical language, and in particular probabilistic language. Although the issues are complex, it is possible to identify some areas for developing an analytical framework to use in Chapter 4. The final section of the literature review focuses on the effects of context on the development of a shared understanding of meaning. Primary school teachers sometimes set mathematics into everyday contexts to provide relevance and to develop dialogue with which all pupils in the class may engage. Practical contexts introduced in classrooms aim to provide opportunity to share meanings, but it seems that quite often there is an assortment of assumptions about the knowledge and experiences the children already have. Developing a shared understanding of meaning is much more complex than teachers may realise.

### **The influence of specific contextual references**

Earlier in this chapter, I reminded readers about my definition of a contextual reference: "A 'contextual reference' is a spoken or recorded reference to an actual *physical situation* or a *mental representation* of such, in which the child or teacher perceives application of the mathematical word." I also included some further discussion of how I considered the use of contextual references to influence the development of a shared understanding of meaning. This section aims to explore and identify different perspectives on **context** most likely to have the greatest influence on developing a joint understanding of meaning of mathematical language.

Chapter 1 included a brief discussion of Rothery's 1980 research that explored the meanings of some mathematical words in 'mathematical' and 'everyday' contexts. For Rothery, a mathematics lesson was a *mathematical*

context, and a mealtime an *everyday* context (Dickson *et al.* 1984 1993 reprint). In mathematics classrooms when children interpret the mathematical word as it is used in an 'everyday' context they might have a different meaning of the word than that intended by the teacher. I wish to explore the issues affecting the process of sharing understanding of meaning through discourse analysis, in order to determine the sense that children make of particular words in mathematics lessons. Teachers often make contextual references, to *physical situations* or *mental representations* of situations, in order to make the mathematical ideas more meaningful for the children e.g. the chance of picking a sweet you like from a box of assorted sweets. In doing so, teachers make assumptions about children's actual life experiences. It seems that there is potential for miscommunication in such circumstances. To inform my research, I consider the various views of context presented by other researchers. First, three definitions of context discussed by Edwards and Westgate (1994), the 'verbal context', the 'context of situation', and the 'context created by the talk', have some relevance. Although I discuss each one separately in relation to the use of contextual references in dialogue, there are varying degrees of overlap. I consider the context created by the talk to be of most importance to inform my analysis in Chapter 4, but each has an influence. As people create talk, they set words into a verbal context, and refer to various situational contexts. Thus, the three views of context become interrelated. Discussing each view separately is a difficult task, but I have attempted to draw out the pertinent issues.

Edwards and Westgate (1994) describe the 'verbal context' as the location of words or linguistic items among other words. Individual words only have meaning by virtue of their relationship with other words (Vygotsky, 1987; Bahktin, 1981). I am considering how the location of the mathematical word within a set of words constituting a 'contextual reference' affects the process of sharing an understanding of meaning. It is possible to take either a semantic or a pragmatic view to researching the verbal context. Semantics is about meaning, usually in relation to lexical and grammatical features, while pragmatics takes into account the contextual, often non-linguistic, factors.

Pragmatics therefore suits my purpose as I am taking into account the various effects of other communicative strategies such as gesture, and the influence of contextual references on developing a shared understanding of meaning in teaching and learning situations. The communicative process will include reference to drawings, practical activities, textbook pages, and various other social events that may or may not help to clarify meaning. The following two sentences illustrate some of the issues about the **location** of a particular set of words within a sentence structure where the contextual reference changes:

1. There is a 50% chance of my coin landing on 'heads'.
2. There is a 50% chance of my team winning the match.

'50% chance' as a phrase means the same in each sentence, but the contextual references change the 'sense' of it. In fact, the second example cannot be true mathematically because there are three possible outcomes to a match. The contextual reference could lead recipients of the message into thinking they understand the meaning of the whole sentence because they understand the phrase '50% chance' and there are two teams in a football match. It could encourage them to think of the number of teams playing instead of outcomes, and lead to a misconception that each team has the same chance of winning a match. Thus, the location of the phrase in relation to the contextual reference can affect the meaning and consequently affect the recipient's response. The verbal context can therefore have an effect on the 'context created by the talk' because it affects the recipient's response depending on the meaning given to the utterance by the recipient. In order to consider the interrelationship between the three views of context I considered the above sentence spoken in different situations:

Situation 1: *Teacher encouraging a child to provide reasons*

Teacher: What chance does your team have of winning the match tonight?

Child: There is a 50% chance of my team winning the match.

Teacher: Why do you say that?

Situation 2: *Two children debating the chances of their own team winning*

Child 1: There is a 50% chance of my team winning the match.

Child 2: No, my team will win it has better players.

Child 1: But we still have a 50% chance because each team starts with 11 players.

Situation 3: *One teacher challenging another's assertion*

Teacher 1: There is a 50% chance of my team winning the match.

Teacher 2: That doesn't make sense because you could draw.

I have changed the situation (context of situation) by altering the participants in the dialogue. In each situation, the same utterance (verbal context) is understood, and responded to in relation to the situation (context created by the talk). Thus, the difficulty with trying to discuss the three views of context as separate entities is apparent. As my work focuses on contextual references that are real or imagined situational contexts, I consider them to be an influential part of the 'context created by the talk'.

Contextual references are usually references to various kinds of situational context, real or imagined. Contextual references to concrete items, drawings or events during the lesson are clearly part of the classroom situational context, while references to imaginary contexts, such as 'playing football in the park' might be part of the children's 'out of school' situational context. I call such references 'imaginary' even though they are real-life situations, as they are not happening in the classroom at that particular time, but children have to imagine the situation in a different place and time. The context of situation is the total setting including everything that can affect the sense that participants in dialogue make of a particular utterance. It encompasses everything physical in the environment, plus everything 'mental' created by the environment. This includes, as part of the teaching, use of pictures, references to real life, stories, use of metaphor, and visual representation of mathematical structures. The situational context is complex and dynamic and participant's individual influence on the situation will affect their future response to any changes. To some degree, language builds upon the social

context in which participants share an array of assumptions (Giles and Coupland, 1991). If this was not so then meaningful interaction could not exist. In different situations, people use different language structures, based upon the social context and the knowledge they have of each other's understanding of the situation. Giles and Coupland (1991) provide an example where a mother who wants her child to put on his new shoes, asks a question rather than giving as command, because she knows that asking a question will achieve the desired result. Power relationships influence the nature of the dialogue in the classroom situation. In order to analyse the dialogue, one has to consider the whole setting, the participants' characters and its objectives. One utterance sets the context for the next and power relationships affect how one utterance builds upon another. Children are not equal partners in dialogue with the teacher (Wood, 1992). Various 'micro' and 'macro-contextual' systems influence, and are influenced by, our use of language (Giles and Coupland, 1991). Of particular importance to my project are the different sets of participant relationships within two different settings. During lessons, the teacher interacts with the whole class, groups or individual children. On occasions, the teacher also interacts with the researcher (me). Children interact with the teacher, other children and me. *The structured interventions provide a different physical setting with three participants, two children and me.* My aim is to provide opportunity for pairs of children to discuss without too much adult intervention, thus providing a different set of relationships than in the classroom. My view is that either the teacher or researcher mainly determines the situational context, but within that situation, there will be various opportunities for the children to create their own context through talk.

Context therefore, is not just the physical setting, but is those things that contribute to the meaning of the talk (Mercer, 1995). Mercer agrees that the talk itself creates its own context as what we say creates the foundation for meaning in the talk that follows. Continuity must also exist alongside the creation of context by the speakers. Continuity refers to the emerging of themes that participants explain, accept, revisit and consolidate. This notion is clearly relevant in mathematics lessons. Recent research by Askew *et al.*

(1997) emphasised the need for the development of a rich network of connections between different mathematical ideas. Initial results from the National Numeracy Project assessments (DfEE, 1999c) seem to confirm that revisiting mathematical themes more regularly and greater emphasis on children explaining their ideas are both factors that ensure sounder learning of mathematics. However, whether children and teachers really do engage in a dialogue that creates its own context is unclear. The emphasis on the teacher as the more knowledgeable partner in dialogue seems prevalent in many primary classrooms. What might become evident over time is that mathematical dialogue is closer to being continuous than fragmented, with opportunity to engage in using the same mathematical language more regularly than in the past.

The 'context created by the talk' is most important for joint construction of meaning. The participants can create the context. They can modify and challenge it through various means of communication. This contrasts the idea of a static set of factors and makes context into a dynamic entity (Edwards and Westgate, 1994). Bakhtin (1981) suggested that dialogic speech entails an active response requiring thought, in which a participant in dialogue reconstructs the social context. This is one way of viewing the context created by the talk. In relation to 'contextual references', the context created by the talk is that which arises when the participants engage in a process of developing a shared understanding of meaning. The speaker describes an event with reference to a particular 'situational' context, real or imagined, and the response depends on how the contextual reference affects the meaning for the listener. Included in the communication process are gestures, writing and drawing to clarify meaning. In my research therefore, a variety of real or imagined situational contexts affect the context created by the talk, and vice versa. Although a particular situational context might be static in nature, once it becomes part of the context created by the talk, it becomes part of a dynamic relationship between language and thought. The following paragraphs present a range of relevant aspects to consider in relation to contextual references, their nature and their effects on developing a shared understanding of meaning. I consider the nature of contextual

references, often as pseudo real-life contexts, as a strong element of the context created by the talk, and a key element affecting the development of shared understanding of meaning.

The lesson perceived as a communicative event, might have particular structures and conventions to which participants adhere (Cazden, 1988). Some teachers might consider communicative competence in the classroom to be the ability to follow the structures and conventions, but not necessarily as the ability to use language to express ideas and make sense of the special language of the subject. For example, in mathematics there is sometimes an expectation that children follow conventional ways of ‘saying’ a calculation method. We might describe this as developing a model for ‘inner speech’ by expressing the thought process verbally e.g. in dividing 450 by 9, (Figure 2.1)

**Figure 2.1 “Saying a calculation”**

Pre 1999 many children would set out a traditional division calculation....

50

9 ) 450

... and say something like this:

“Nine into four won’t go”

“Carry the four”

“Nine into forty five goes five”

“Nine into nought goes nought”

The National Numeracy Strategy encourages a more flexible approach to similar calculations, and most children will begin to say sentences such as, “Nine fives are forty-five, so nine fifties must be four hundred and fifty.” Interestingly though, the training materials for the NNS include an example of what children should say to themselves when doing a standard algorithm, in a clear attempt to help the child have an image of the numbers involved with due regard to place value (Figure 2.2). My particular interest is in the fact that flexible mental calculation strategies should underpin written algorithms as recommended by the Qualifications and Curriculum Authority [QCA] (1999). The children express their mental strategies in their own



words and develop a mathematical vocabulary through dialogue with the teacher. Some teachers might unnecessarily impose a particular language routine for a compact method. If the children have secure mental methods, their own ways of expressing a written method might be more appropriate. There seems to be an element of discord between the interactive social context that enables mental strategies to develop through a flexible dialogue, and a didactic social context that appears to impose a linguistic structure. The imposed structure may or may not be similar to the child's personal model, and contradicts Hicks, (1996) idea that learning is a response to the meanings derived in a particular setting.

### Figure 2.2 Language used in compact method

$$\begin{array}{r} 547 \\ + 276 \\ \hline 823 \\ \hline \end{array}$$

1. Seven plus six equals thirteen, write down three and carry ten (child writes carry digit underneath)
2. Forty plus seventy equals one hundred and ten, plus the extra ten, which equals one hundred and twenty. Write down twenty and carry one hundred (child writes carry digit underneath)
3. Five hundred plus two hundred, plus the extra one hundred, which equals eight hundred.
4. The total is eight hundred and twenty three

(DfEE, 1999d, page 35)

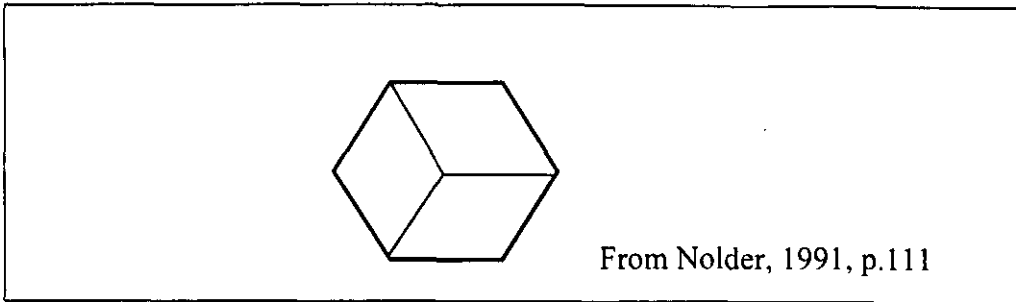
It is especially important that when teaching mathematics we achieve the correct balance between children expressing their own ideas and teaching them routine language conventions. I believe that encouraging flexible mental methods of calculating encourages the development of a flexible mathematical language that children can use to 'create their own context' through which they develop a shared understanding of meaning. In contrast, the teaching of routine 'technical terms' or mathematical words, in the past has been problematical as shown by various pieces of research (Mercer, 1995). Mercer suggests that teachers can introduce technical terms into

dialogue with pupils where the context helps to make the meaning clear. He cites an example from a science lesson in which the teacher introduced unfamiliar words during a discussion about an experiment presented in Edwards and Mercer, (1987). The children began to use the words in their discussion, following the teacher's model. Whether they had developed a shared understanding of meaning is unclear, but they used them in context replacing previously used everyday words.

An important part of the context in teaching and learning is the use of gesture to exemplify meaning (McNeill, 1985). McNeill wrote about language viewed as action, and he described a mathematical discussion in which gesture was important in indicating a change of course in meaning. Gestures help the listener develop images about the words, and the mathematics involved. Unfortunately, many gestures are spontaneous creations possibly leading to confusion of meaning. Primary teachers commonly use the words and accompanying gestures for 'higher' and 'lower' when discussing *values* of numbers. Such gestures might accompany a visual stimulus such as a 'hundred square', which most often has the numbers 1-10 across the top and the numbers 91-100 forming the bottom row. To the children the 'lower' numbers (in value) are 'higher' than the others (in position) and vice versa. The children might focus on position rather than *value* because previous gestures and use of words might create a positional image. The words 'worth more than' and 'worth less than' might be better for describing the *value* of a number. My preference is to consider gesture part of the context created by the talk because the gesture, alongside the words used, affects the development of a shared understanding of meaning, as described in the above example. I might classify it as part of the verbal context in relation to the gesture helping to locate a word or group of words among others. As part of the situational context, it becomes an external influence on the dialogue rather than 'language viewed as action' suggested by McNeill (1985). However, as part of the context created by the talk it helps to exemplify the meaning of words and phrases and is 'language viewed as action'.

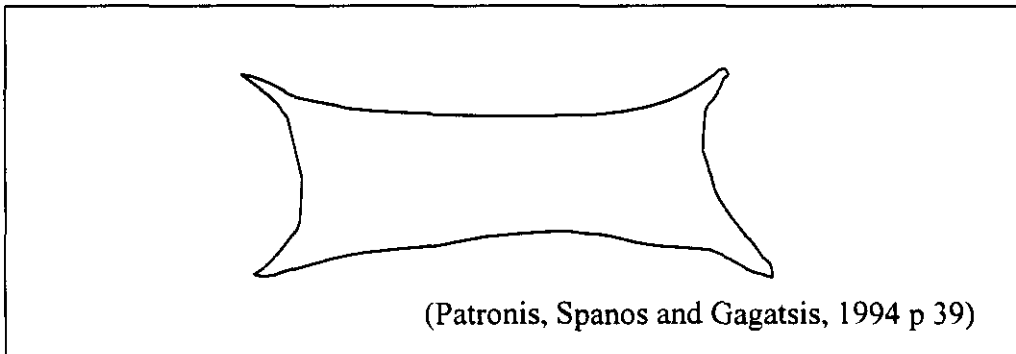
The following paragraphs consider the various ways that pseudo real-life contexts influence the teaching and learning process. Following my previous debates about the nature of such contextual references, I view them as part of the context created by the talk because of their direct influence on the communication of meaning. Teachers sometimes use stories to embed a cognitive task in a meaningful context (Donaldson, 1978). However, the images and language that such stories use might be inconsistent with children's everyday experiences (Walkerdine, 1990 reprint). Walkerdine questioned the relationships between the Bear family members in "Goldilocks and the Three Bears", as they often reflect sizes that increase in direct proportion, and have clearly defined father, mother, and baby relationships. In real families mother might be taller than father might, and father might be the 'cook'. The words big, bigger and biggest are not specific enough. A person, and a 'Bear', or their bowls, chairs and beds might be 'bigger' than another, in a variety of dimensions might. Walkerdine (1990) believed the 'meaningful context' in this case to be potentially confusing for children with reference to the mathematics involved. It is possible that the story discussed above, can become a metaphor for ordering three objects by size in a child's mind. The use of metaphor is sometimes a useful tool in mathematics. Teachers express metaphors using all the senses, with media depending on the age group and mathematical content. A metaphor uses the notion of 'seeing the mathematical idea as something else' e.g., an equation as a balance (Nolder, 1991). However, this can lead to children using metaphors from their own experience that lead to potential confusion. Nolder (1991) described some children looking at a stained glass window design to identify polygons. One child 'saw' a square. Others did not at first (Figure 2.3 overleaf). There was not a square in a two-dimensional sense, but the child actually visualised a cube in the design. A cube has square faces and to describe it the child used 'square' because the task was to identify polygons. Square became a metaphor for cube in this context. This has implications for ideas that develop in one context and their transferability to other contexts.

**Figure 2.3 Rhombuses, a cube, or a square?**



Other research highlights the power of the perceptual image compared to the specialist language of mathematics. Thirteen-year-old children drew incorrect convex quadrilaterals (Figure 2.4)

**Figure 2.4 A convex quadrilateral?**



Patronis *et al.* (1994) believed the children interpreted 'convex' in the non-mathematical sense. They concluded from their research that children's ability to classify shapes relied mostly on their 'perceptual meaning' rather than the abstract logical meaning derived through knowledge of the language describing the properties. The power of the visual image is something that teachers exploit, particularly with young children, but it seems caution is necessary. Such images might not transfer easily into the more abstract situations. Some researchers such as Ball, (1993); D'Ambrosio and Mewborn, (1994); Nunes and Bryant, (1996) claim that particular representations of mathematical ideas obscure the development of other ideas. In my own research, the probability scale is potentially a confusing image of a continuum from 'certain' to 'impossible'.

Pendlington (1999) asserts that developing imagery is an important feature in developing mathematical ideas and children might develop various types

of images to use in different mathematical situations. Part of the teacher's role is to attempt to help the child build such images by providing a range of sensory experiences. Sometimes, pictorial images cause confusion because of their presentation. It is important to ensure that such images focus on the mathematical concepts and relate clearly to the associated vocabulary (Harries, *et al.* 1999). Harries *et al.* identified a number of "distractions or interferences" in some United Kingdom primary maths texts that affect the sense that teachers and children make of them. These include colour and pictures with no significance, poor layout and most important a lack of clear teaching instruction for the mathematics. Ollerton (Hawker and Ollerton, 1999) questioned the desirability of setting test questions in 'real-life' contexts. Most recently, research into the responses to SAT questions at KS2 (Key Stage 2) and KS3 has shown that working class children are disadvantaged because they cannot relate to the 'realistic' contexts in questions (Cassidy, 2000). The situation distracts them from the mathematics. One of my concerns about texts is that the 'contextual references' within them are not always immediately relevant to the lives of all children. A second concern is that some texts promote 'modelling' of both the mathematics and associated language as the main teaching and learning approach. Both these concerns affect transferability of learning across contexts. I recognise that one might consider such resources part of the situational context, but my interest is in the way they affect the development of thought and dialogue i.e. the context created by the talk. In this sense, the texts and pictorial images themselves lose their situational impact as the contextual **ideas** they promote become internalised through dialogic exchange.

Choosing a particular representation of an idea and presenting this as a single model, might eventually limit opportunity for the child to make sense of another situation. It is important that the mathematical knowledge gained is not context-bound i.e. it must transfer to other contexts (Tirosh, 1990). A typical approach to teaching probabilistic language in British schools is to introduce a probability word scale. The current NNS Framework for Teaching Mathematics (DfEE, 1999a) teaching programmes for Years 5 and

6 include the development of probabilistic language and concepts:

Year 5 - Discuss the chance or likelihood of particular events.

Year 6 - Use the language associated with probability to discuss events, including those with equally likely outcomes.

The emphasis is on teaching children the language involved. The accompanying vocabulary book lists the various words and phrases to teach the children. The Framework includes 'even chance' as a new phrase along with 'equally likely', 'equal chance', 'fifty-fifty chance', 'biased' and 'random' in Year 6. Exemplar material shows a probability scale for Year 5 that includes 'even chance' while the probability scale for Year 6 names the same point as 'evens' or  $\frac{1}{2}$ . Thus, the images are different from the proposed content, even in NNS documentation. The most important point to make about the nature of these exemplary materials is the statement made at the top of each column:

"As outcomes, Year 5 pupils should, for example:

Use, read and write, spelling correctly....."

(DfEE 1999a, Section 6, Page 113)

The exemplary material suggests that children should be able to discuss probabilities, and to match probabilities to a scale. There is no specific statement about developing **understanding** of the language, or the mathematical concepts. Understanding the concept of probability is included in Year 7 where a key objective states that children should:

"Understand and use the probability scale from 0-1: find and justify probabilities based on equally likely outcomes in simple contexts"

(DfEE, 2001, Section 3, page 7)

The emphasis before Year 7 is on children learning the language in experiential learning situations. The purpose of the probability objectives in

the Year 5 and 6 programmes is to prepare children for achieving the stated key objective in Year 7 when they begin to calculate simple probabilities. Everyday examples occur in the Year 5 exemplars, while the exemplars in Year 6 move towards a more structured approach using dice and coins to discuss equally likely outcomes. Typically, the most recent primary mathematics schemes tend to use the probability scale, and will often suggest that children place an assortment of everyday situations on the scale. The only change since the introduction of the National Curriculum in 1989 is the higher recommended age for introducing probabilistic language. The original documentation included probability in the Key Stage 1 curriculum.

So far, I have discussed a range of different perspectives on 'context', how different writers describe it, and its effects on teaching and learning. In the previous section of the literature review, I introduced constructivist views about the teaching and learning of mathematics, and the use of 'natural language' to explain mathematical ideas. In my view using natural language to explain mathematics links well with the notion of setting mathematics in 'everyday' situations, although the two do not necessarily go together. 'Natural language' affects the learning of mathematics (Pirie, 1997). This language can have changing meanings with different contextual references made in mathematics lessons affecting children's constructs. The exploration of 'contextual references' in which children perceive words applying may demonstrate how contextual relationships made during the teaching of a topic limits the broader development of understanding of the language involved. I have previously referred to the idea of continuity, and that current British primary mathematics teaching programmes might help to maintain continuity of dialogue in the sense that the class revisits the same language on a regular basis. However, I have not yet identified any research that clearly indicates any effect of the National Numeracy Project on children's learning of mathematical terminology.

## **Summary of Chapter 2**

The first section of the literature review discussed the need for a specialist language in subject teaching and put forward contrasting views. Some

people believe that a specialist language is necessary and there is evidence that children who fail at school are those who fail to take on the 'school meaning' of the language involved. Other research demonstrates that knowledge of a specialist language is not necessary for conceptual development in the subject and suggest that the ability to engage in mathematical dialogue is more important than knowing the meanings of particular words. The current British trend in mathematics education is to encourage dialogue including the use of specialist terminology.

The second section discusses the role of language in learning and considers how words need to be set into a phrase or sentence in order for them to make sense. Bakhtin (1981) placed most importance on the dialogicality of utterances with each utterance building on the sense made of the previous one. Vygotsky's (1987) notion of 'inner speech' and Kieren's (1993) 'image making' and 'image having' are important processes in making sense of mathematical concepts. The role of the adult or other more capable person is highlighted in discussion of Vygotsky's (1994) 'zone of proximal development'. Social constructivist research focuses on observing conceptual development through the child's 'natural language', while other research recognises that children gradually refine their language and use mathematical words. Teaching styles and types of talk in the classroom affect the learning process and there seems to be little opportunity for 'educated discourse' in many primary classrooms.

The final section clarifies the meaning of 'contextual reference' within the scope of this project and discusses different views about the nature of context. The verbal context is the location of the word among other words. This is particularly evident in the structured interventions when the children will place the word within a sentence that will most likely contain a 'contextual reference'. The context of situation is the whole setting of which many interrelated aspects might affect communication. The relationship between participants and the contextual references that derive from various situational contexts are of most relevance as these become part of the context created by the talk. The most relevant context is that created by the



talk itself as the participants make sense of each other's utterances. However, I also include within this category the influence of gesture, modelling linguistic procedures and effects of perceptual images. Finally, I summarise the expectations of the National Curriculum for children in Years 5 – 7 in learning probabilistic language as outlined in the National Numeracy Strategy. This forms the broad national context in which the teacher and children work to develop a secure probabilistic vocabulary. The analysis will draw upon relevant issues from each view of context as appropriate.

## **Chapter 3: Research Methods**

This chapter has five main sections. The first describes the setting in which my research took place. The second section begins with a discussion of relevant research methods for researching classroom talk. Following this, four sub- sections discuss specific issues pertinent to the two strands, and other particular aspects of my chosen methods such as the use of dialogue as evidence and discourse analysis. I include relevant detail of the pilot study conducted in fulfilment of the Educational Doctorate Stage 1 in the third section. I designed the pilot study to identify the most effective data collection methods, and refine the research focus so I discuss its impact in the fourth section. For Stage 2 EdD, the methods I used are appropriate for a part-time researcher working on a two- year project. Finally, I outline my final research phase methods before summarising the whole chapter.

### **The setting**

In order to gain access to a school setting I relied on finding a teacher with whom I had a good relationship and who would allow me to observe in the classroom. Thus, I did not choose the setting as representative of other settings, but I negotiated to research a previously unknown setting through a known contact. Each educational setting is unique although different settings will undoubtedly have similarities due to the nature of curriculum content and current trends in teaching. For my particular research project, the type of primary school and its national levels of attainment are only relevant if someone wished to carry out a comparative study elsewhere. I have not set out to make judgements about mathematical or linguistic ability, nor do I intend to make comparisons between different attainment groups, or socio-economic groups. What I learn about communicative processes, and the use of contextual references to exemplify word meaning will be relevant to any mathematics classroom situation. The potential for miscommunication and failure to develop a joint understanding of meaning is present in every educational setting. Thus, every teacher needs to become aware of the issues, and consider means by which to address them within their own classroom.

The fieldwork took place during the school year September 1997 - July 1998. The large Catholic primary school, in a suburban seaside town in the North West of England, had approximately 500 children. The area is predominantly middle-class with the majority of the housing privately owned semi-detached and some garden-fronted terraced housing. Each year group had three classes. I studied one Year 5 class (ages 9-10), of 31 children, where the teacher was newly qualified, and used to people observing his teaching. He was also a mathematics specialist, and interested in my project because he felt he had little knowledge of how children learn mathematical language. The class sat in attainment groups during mathematics lessons, according to previous assessments, progress through the mathematics scheme, and the amount of teacher support required. A plan of the classroom shows the seating arrangements (Appendix 3). My research did not make comparisons between the attainment groups, although I used them for organisational and identification purposes. The teacher also paired the children for the structured interventions based on their classroom groups. His justification was that the children would be more confident working with a known partner, than with a new one. I accepted his judgement, because I had little knowledge of the children and it was of prime importance that they felt comfortable with the situation.

The school used the Heinemann mathematics scheme (Scottish Primary Maths Group [SPMG] 1994 and 1995 revised edition), drawing from text materials as needed. It was a time of impending change, with the National Literacy Strategy starting in September 1998, and the National Numeracy Strategy in September 1999. The teacher used the National Numeracy Project (NNP) vocabulary book to help him determine which mathematical vocabulary Year 5 should learn. The Local Education Authority advisory service promoted these books during training on the NNP at the school. A display of mathematical words was up in the classroom, again following LEA advice in teaching children mathematical language. Although the whole class were present during the lesson observations, a smaller sample of children were involved in more detailed structured interventions. These took

place either in the library or in the corridor at the top of a flight of stairs because I had to fit in with the school timetable and room availability.

### **Methods for researching classroom talk**

Pirie (1997) outlines three commonly used approaches to researching language. First, to '*examine talk itself*', i.e. examining the words children use and the meanings they give them. Secondly, '*exploring talk as a medium through which teaching and learning can happen*' which includes all aspects of classroom practice, including the role of teacher talk. Her third approach, '*analysing talk to explore other classroom events*' involves analysing the roles, relationships and responsibilities within the classroom situation. Pirie (1997) describes this as consideration of power relationships and children having the responsibility for their learning more firmly in their grasp. In any research, all three approaches have relevance, but the emphasis might fall on one approach more than the others might, at different times. For my research, I shall *examine the talk itself*, to identify the contextual references made in explanations. However, my examination will go beyond this into *exploring talk as a medium for teaching and learning*, using discourse analysis to identify how the teacher and children develop a shared understanding of meaning of mathematical language. Within this, I will consider the *effects of power relationships on the talk*, and vice versa. Pirie (1997) does not outline any particular methods used in these approaches but it seems logical to assume that for each one, collecting examples of verbal and non-verbal communication is essential.

There is danger in adopting a methodology just because it is currently popular, or accepted by a particular 'research community'. Much recent mathematics education research appears to have its roots in constructivism, either radical or social, but with a trend towards social constructivism (Ernest, 1994). Steffe and Tzur (1994) conducted teaching experiments to analyse social interaction when children work at the computer and challenge the radical constructivist view that assumes learning is univocal. After working with the children, and analysing the dialogue in relation to the computer screen context they put forward the view that mathematical

learning is dynamic and reliant on social interaction. They researched how children constructed mathematical understanding and considered the type of teaching situation needed for secure learning. Constructivist research by Saenz-Ludlow (1995) also involved a teaching experiments and individual interviews with the children. The purpose of Saenz-Ludlow's (1995) research was to discover how children constructed understanding of fractional concepts. Saenz-Ludlow (1995) spent some time as an observer in the classroom before choosing six children with whom to work. Clinical interviews with each child ascertained their current ways of operating with number, and to a lesser extent, fractions. A series of lessons with the researcher followed the interviews, with the explicit goal of fostering the generation and evaluation of the child's conceptual constructs. As in Piaget and Inhelder's (1975) research, language was a tool for children to achieve and express cognitive processes. Saenz-Ludlow (1995) did not focus on the communicative processes, but on interpretation of a child's conceptual development as part of a teaching and learning episode. This 'experimental' approach to research is not useful to me because I plan to observe natural teaching and learning situations with as little intervention as possible. Research methods must suit the research questions, so in order to answer my first research question "*How does the teacher interact with the children to develop a shared understanding of the meaning of mathematical words?*" I must observe actual lessons. For my second research question "*How do children express their understanding of the meaning of specific mathematical words and phrases?*" I could adopt an interview technique with a specific set of questions. However, there is the possibility that an interview will promote a particular style of answer from the children. The responses might be presentational rather than exploratory as described in Chapter 2 with reference to Barnes (1992). I aim to provide an opportunity for some exploratory talk to develop through 'structured interventions', which are described more fully later in this chapter, and I plan to include all aspects of communication, verbal and non-verbal in my consideration of how children express their understanding of meaning. Children explaining meanings of specific mathematical words to their peers will provide opportunity for me to observe and collect appropriate data.

Qualitative research methods are most appropriate for researching the ways children and teachers share their understanding of the meanings of mathematical vocabulary. The main features of qualitative research are:

1. A focus on natural settings;
2. An interest in meanings, perspectives and understandings;
3. An emphasis on process;
4. A concern with inductive analysis and grounded theory;

(E835 Study Guide, p85)

Quantitative methods are inappropriate for the small sample of lessons observed and the number of children I work with. I have not chosen to test a theory, nor am I looking for causal relationships in terms of independent and dependent 'variables' as in a scientific situation. I am interested in developing theory through interpretation of communicative events. I believe that the analysis and conclusions drawn from any research, including those with a strong quantitative basis, rely on interpretation. In quantitative research, this often requires interpretation of measurements, while in qualitative research there is often interpretation of observed events, including communicative events. Graue and Walsh (1998) provide an interesting discussion about the importance of interpretive qualitative research when children are the objects of inquiry. I agree with their claim that much of the world is not readily measurable and that a good narrative description is often more accurate than a measurement description. The strength of measurement is precision rather than accuracy. The approach suggested by Graue and Walsh (1998) involves the researcher working face to face with a few subjects over an extended period. This approach seems particularly suited to a primary practitioner who is normally in daily contact with children. Primary classroom teachers spend much of their time interpreting events such as children's responses to questions, in order to inform their teaching. The National Numeracy Strategy (DfEE, 1999a) considers such informal formative assessment to be the most important assessment process because it informs day-to-day teaching. Thus, it is quite natural for primary educators engaging in research, to feel confident about

their powers of interpretation. An example of how such interpretation influences subsequent thoughts and events is the development of my hypothetical ideas about contextual references. Although I use the word hypothesis, I must emphasise that I am not using it as a theoretical standpoint that my research aims to support or refute. I am more concerned with developing theory through presenting initial ideas, and then readjusting my ideas to form new hypothetical ideas in an iterative process. My hypothesis about contextual references, 'that contextual references most likely arise from the speaker's 'imagined' situational context that they perceive the word to describe', as discussed in Chapter 2, was a 'vague idea' initially based on a collection anecdotal experiences over a period of several years. I had noticed that children often did not give the same meaning to words and events that teachers intended. Initially this occurred in my own classroom, and more recently, I observed this happening with others. It is wrong assume that the children misunderstood the teacher without considering why the communicative process was unsuccessful. In fact, as a classroom teacher, I refined and improved my communication skills, through reflecting on such observations. However, I have also recognised that not all teachers reflect deeply on such events, preferring instead to assume that the children have not listened properly. With the opportunity to conduct pilot research, my 'vague idea' became more clearly defined as an issue within teaching and learning. Further research and analysis will provide the grounding for developing theory about the influence of contextual references on the development of a shared understanding of meaning. However, I cannot prove or disprove my ideas about the thoughts of another person, because I am reliant on making judgements about their thoughts based on their communications. The reliability of my findings will depend on the quality of my methods, and the relationship between all participants. As my research has two distinct strands, I shall discuss the specific research issues for each one although some general ethical and methodological issues, such as the implications of using a video camera, are common to both. I discuss such issues as they arise in each strand.

### **Whole-class lesson observations:**

Using one class, in one local school limits the validity and the generalisability of my findings. However, teachers reading the results of such research often recognise similar situations arising in their own classrooms nationally, and internationally. Such recognition validates the findings of small classroom-based projects and highlights the relevance of researching a few cases in depth as suggested by Graue and Walsh (1998). I recognise that a newly qualified teacher is not representative of the teaching population, but observing an inexperienced teacher will provide evidence of the communicative challenges when developing a shared understanding of meaning, perhaps more so than observing an experienced teacher. Any teacher who allows a researcher in the classroom and has an interest in the language of mathematics is also unrepresentative, and so one can justly claim that the lesson observation situation is not 'natural', although it is more typical than an experimental teaching episode as in the previously discussed research by Steffe and Tzur (1994) and others.

The classroom was the most natural setting, in the sense that it was where the children normally spent their lesson time. The lessons were typical mathematics lessons for a primary school that is beginning to follow the three-part lesson structure of the National Numeracy Project. The teacher and children knew I was focusing on the teaching and learning of mathematical language. From past experience, I know that teachers feel threatened by an observer's presence when the focus of the observation is unknown. This is an ethical issue, and I agree with Graue and Walsh (1998) that all participants, ought to know exactly why the research is taking place. When working with children a researcher often takes the place of a teacher, but does not always behave exactly as a teacher does (Graue and Walsh, 1998). It is therefore important to ensure everyone is aware of any differences in approach. Parents might be concerned if they thought the research activity was hindering their child's learning. Before my research began, the school informed parents, on my behalf, about their children's involvement. Parents had the opportunity to refuse involvement on behalf of their child. I gave assurances that all recorded material was for personal



research purposes only and that my work would maintain the anonymity of the children and the school. In short, being ethical requires being honest to all involved (Graue and Walsh, 1998).

Reactivity is a risk when everyone knows the purpose, and the teacher might use a different approach to teaching, by providing lessons with language as a focus. Children might make more effort than usual to use particular types of language thus creating a biased collection of data. However, from my previous experience, any effects of sharing the research purpose are minimal, and if mathematical language comes into the lesson more than usual, it provides more opportunity for me to observe it. The reliability of such data depends on analysis that accounts for any possible effects. My lesson observations focus on the 'educational discourse', which encompasses all communication between participants in the pursuit of learning (Mercer, 1995). The strength of a qualitative approach is that natural settings provide detailed data about specific incidents allowing detailed analysis of the relationships between people, time, places and events. I accept that a researchers presence will alter the 'normality' of the situation and that the teacher and children might react to it in a variety of ways. Edwards and Westgate (1994) cite Wragg (1984), who noticed that teachers alter their behaviour simply by being irritated that someone is watching their every move. However, establishing a working relationship with the class before the observation phase will help to reduce reactivity. A non-participant observer may remain detached, but it is still possible to misinterpret events (Wragg, 1994). Teachers have to make speedy decisions when responding to new information brought to the lesson by a child. This might cause tensions between keeping to lesson objectives but being flexible enough to recognise opportunities to provide extension or simplification. The observer must remain alert to such tensions and focus on actual events without making immediate judgements. It is therefore essential to find out the teachers lesson objectives, and to discuss with the teacher after the lesson the reasons for any change. Classroom educational discourse comprises of "long conversations" with a history and a future (Mercer 1995). This notion led to my decision to observe three lessons that

developed a mathematical theme. Not all classroom language deals with developing concepts, as teachers and children have to talk about practical organisational matters. These are all part of 'making sense' of the whole situation for the child. It is possible that the child can make sense of the mathematics, but not the actual task required. As discussed in Chapter 2, Vygotsky (1962) said that we only understand another's thoughts when we understand the reason for their communication. He is suggesting that we need to know what has motivated the child to say what they did. In my research, this means knowing the previous contexts in which the child has experienced that mathematical concept, and the language used. Three consecutive lesson observations provides opportunity to follow the process of sharing understanding of meaning and will help to identify the effects of particular contextual references over time.

Having made a decision to explore a classroom setting it was necessary to consider the range of means of collecting information, as unobtrusively as possible. Suitable methods might include having a form of systematic observation, such as a coded checklist, or a time-interval sample. Edwards and Westgate (1994) discussed the issues and I agree with them that systematic observation provides many data, coded and quantifiable, but is most useful in large research projects, with several observers involved. As I am working alone in a single classroom on a small-scale project such an approach seems unwieldy and quantitative data unnecessary to answer my research questions. In a natural setting, where the research aims to develop theory rather than test theory, using any form of code, or timed observations is inappropriate. Both methods might lead to important data being missed that might otherwise be captured by other more continuous methods. A pre-conceived coding would require me to make assumptions about the nature and content of the interactions. Having discounted this approach as unsuitable, the choices were narrative, audio and video recording. I was interested in narrative recording, as I thought this the least unobtrusive, but probably difficult to record all events in enough detail and accuracy. At the other end of the continuum, I considered the video recorder the most useful for collecting a lot of detail, but also the most likely to cause effects of

reactivity from participants. The following paragraphs discuss some possible issues.

In Pimm's (1997) classroom research, on mathematical language, the technique of 'eavesdropping' proved successful. 'Eavesdropping' involved him sitting out of direct sight of pairs of children working on computer problem-solving activities. He did not involve himself in conversation with the children at all. In this way, he was able to listen to children's talk and record it clearly in note form, with minimal distraction. The purpose for this method was to discover the exact nature of children's language when working in pairs within a computer environment. The method suited the purpose. 'Eavesdropping' might be a useful approach for me when listening to groups of children working together, in the classroom. However, it might be necessary in my research to ask questions to discover a child's thoughts about a particular word. Not all classroom talk will necessarily use, or refer to the mathematical words I am researching. A strategically placed video recorder might also be a suitable tool for 'eavesdropping', but Pimm (1997) disagreed with the use of video recording because a video recorder cannot collect other details that a person might notice in the situation. In the case of two children working at the computer for example, the researcher can move discreetly to look at the screen, but in most classroom situations, one cannot move a video recorder without distraction. Thus, essential 'contextual' information might be lost. In Pimm's (1997) research, he needed to know the computer screen contents to link it to the dialogue, in order to set it into context. Use of video recording also might lull researchers into a false view that the re-visiting of the video record will enhance the understanding of the situation (Pimm, 1994). I tend to disagree with Pimm and think there are benefits to using both direct observation and video records in tandem. Hicks (1996) suggested that each method has its advantages and disadvantages and by combining the different approaches to researching children in classrooms we gain more relevant information than if just one method is employed.

From my own experience, being part of the lesson as an observer provides one perspective of the situation, but when watching a video recording I pay

attention to those aspects I did not pay attention to **during** the observation period. Video records of personally observed situations allow retrospective analysis that can only enhance other information collected (Edwards and Westgate, 1994). However, I agree with Pimm (1994) that we cannot ever truly know the 'whole' situation and what it means to each individual participant. Watching video recordings will clearly identify any differences in discourse, between children's responses in a natural setting and those in a 'modified' setting, while during observation one might not notice such things (Walkerdine, 1990 reprint). For the purposes of discourse analysis, and for my specific purpose of discovering the teacher and children's communicative processes when they share their understanding of the meanings of mathematical words, use of video recording is essential. A researcher cannot possibly make handwritten notes of the exact dialogue and other communicative events over a sustained period, although handwritten notes are useful for short exchanges, and other off-camera events. Audio recording is generally less obtrusive, but contextual detail such as facial expression and gesture is lost, unless the observer makes careful and copious hand written notes (Swann, 1994). Researcher bias towards a particular style of teaching might also affect lesson observations (Lacey, 1993). By recording actual events, and not making immediate judgements, a more objective approach ensues. All the information gathered will generate a working hypothesis. As the research progresses there is a need for the researcher to develop his or her powers of discernment to select specific aspects for scrutiny during observation (E835 Study Guide p90). A gradual focusing and refocusing of attention should occur during observation, tape transcription and analysis.

The possibility of reactivity due to a researchers presence or video recording means that researcher judgements require participant validation. Involving the teacher in either a post lesson interview or questionnaire will provide a means of triangulation to allow judgements based on observations to be clarified by the participants' own perception of the situation (Hargreaves *et al.* 1975, E835 Study Guide p98). A semi-structured interview is probably most appropriate, especially if it is taking place immediately after the lesson.

The skeleton structure allows both a line of enquiry and flexibility to follow up responses in more depth. An interview at the end of the lesson allows for immediate recall of events by the teacher, but also puts constraints on the researcher in developing questions quickly, based on classroom observations alone. This might lead to missed opportunities to explore an idea that develops after further reflection through reading one's notes and watching video-recordings. Viewing the video recordings before constructing questions enables a clearer focus on specific issues. Some questions will only arise after a significant amount of analysis. Opportunity for the teacher to watch the video recordings might also be useful, because the teacher will gain a different perspective of his or her own actions and have more opportunity to reflect before providing answers. Such reflection can also be promoted by providing a questionnaire rather than holding an interview. The act of writing answers also enables thoughts to be fine-tuned. In both situations, devising questions to elicit particular information is the main challenge. Ambiguity is often a problem, particularly in questionnaires. In an interview, the researcher can clarify the ambiguity, but for questionnaires completed away from the researcher, clarification might be more problematical. Whatever method of participant triangulation is chosen the essential feature is that communication is clear enough to elicit the information required, but flexible enough for either party to approach the other for clarification.

### **Structured interventions:**

The structured interventions took place in a different setting outside the classroom and the activity was a valid educational activity, but not typical of primary school classrooms based on my own experience and knowledge. Children also had the option to not participate in the structured interventions if they chose not to, as is their right to do so. Earlier in this chapter, I explained that I had chosen to organise structured interventions with the children, rather than use interviews. A structured intervention is not a teaching episode, nor is it pure observation of children at work. It is an activity structured specifically to elicit the type of information required by the researcher as the children work together. At the same time, it allows for

intervention by the researcher, at appropriate moments, in order to elicit a specific piece of information. The aim of a structured intervention is to provide opportunity to observe peer communication, with a specific focus, in a pseudo-natural educational setting. A children's activity sheet is a useful guide to provide a structure and opportunity for written or pictorial records that may or may not arise out of the discourse. Other researchers have found that collecting data as pictorial images helps to explain a child's thoughts, and they are an integral part of the research process (Hicks, 1996; Patronis *et al.* 1994; Pa, 1991). Children use their imagination to explain and justify their thoughts and choices to others (Pa, 1991). The activity sheets in my research (Figure 3.2) are linguistically productive because children have to describe, explain and reason in their own words. They are *cognitively demanding because the children are trying to demonstrate how they share their understanding of the meanings of mathematical words.* Finally, the main section is contextually unsupported other than the opportunity to use a pencil and paper to aid thought processes. My justification of the method is that it allows exploration of the children's thoughts through interpretation of their communications, without imposing models on them. Explanations children give might reflect the teaching they had, the types of contextual reference made during the lesson, or personal perspectives.

I also provide an opportunity for children to justify their choice of true or false response to a statement, which includes contextual information, after the main section is complete. My reason for including this is to observe the types of language children use and the types of exchange different pairs enter into, about exactly the same contextual reference. In expecting children to explain their reasoning, to enter into a dialogue and question each other the structured intervention explores whether a child can take part in 'educated discourse' (Mercer, 1995). Mercer writes about 'educated discourse' as new ways of using language to think and communicate. An important feature of such discourse is the ability to make ideas accountable to a body of knowledge. In order to elicit ideas from the children I adopted a pseudo-teacher role in providing and monitoring the activity. It was different

from other classroom activities because of the requirement for children to enter 'educated discourse'. There is justification in claiming this because in many primary classrooms teachers 'educational discourse' does not include 'educated discourse' (Mercer, 1995).

### **Dialogue as evidence:**

Data collection is an important methodological issue, but just as important is the nature of the transcripts that document the lesson's character to identify the nature of interactions that occur (E835 Study Guide p189). One must remain objective in recording classroom events (Wragg, 1994) and a video camera helps to retain objectivity by providing a record to revisit and check compatibility of the dialogue with classroom events when transcribing. It is important to add information to transcripts of dialogue such as perceived communication and control relationships that will provide depth of analysis later (Edwards and Mercer, 1987). Thus, transcription of tapes must convey contextual evidence considered relevant to the research. My interest is in the nature of the interactions rather than the structure of the conversation. On reading research that relied heavily on transcription of conversation (Wright, 1993 pp. 23-54) the transcripts, although containing contextual clues, still left me wondering exactly what the classroom situation was. The context of situation could have influenced children's responses, as could tone of voice and facial expression. Observations, or other comments, noted immediately or on later transcription and analysis of tapes might be included in an extra column (Swann, 1994). Based on my pilot study experience I decided to use a standard format with a column for actual observations, and a separate column for my comments and thoughts to aid analysis (see Appendix 4). There is value in such information, but I must aim for impartiality. A written transcript and video record allow retrospective analysis of events that one has also seen in action. It is useful to combine the two forms of enquiry in order to interpret events more fully (Hicks, 1996). Hicks' claims that both methods are acceptable forms of inquiry into discourse, and writes about multi-layered interpretations. Thus, one can interpret events as they happen, but then revisit the video recording, written notes, and transcripts of the event and reinterpret in more detail.

### **Discourse analysis:**

Different writers describe discourse analysis in different ways, depending on the situation, and the discipline that forms the basis of the enquiry. Using ideas from a range of different sources, I shall describe my method and the justification for it. Discourse analysis is necessary in both strands of my research, possibly requiring slightly different approaches, although the principles will remain the same. In the classroom observations the focus is on dialogue between the teacher and the whole class, while in the structured interventions the focus is on peer interaction with researcher intervention. My aim is to identify the ways that both types of dialogue contribute to participants shared understanding of meaning of mathematical vocabulary.

Edwards and Westgate (1994) set the research methodologies into an historical context, providing a useful overview from which to draw ideas. They suggest that analysis of classroom discourse requires sensitivity, and should provide a linguistic account of the processes involved in teaching and learning. During discourse analysis particular speech acts might be categorised retrospectively. In this way, the researcher responds to the data, rather than fitting the data to the categories. This approach suits my own purposes, yet I am also aware that once I have decided on my categories there might be instances when the dialogue does not fit the category and variation occurs (Potter and Wetherell, 1995). The idea of categories does not seem to fit with detailed interpretive studies of individual and unique construction of meaning (Hicks, 1996). However, I think that there is a place for attempting to categorise some types of exchanges described by other researchers with some modification to suit my research. At the same time, I wish to convey the particular influence of contextual references in the process of sharing understanding of meaning.

Classroom discourse is all the language used during intellectual and social activity in the classroom (Mercer, 1995). As previously identified one main purpose of classroom discourse is to use educational discourse in order to develop the children's ability to use educational discourse. Schiffrin (1994) lists a set of underlying principles to guide the analysis of discourse that she



considers all analysts, regardless of their particular discipline, ought to follow. The list includes identifying coherence in the discourse in developing meaning. Coherence occurs when participants produce utterances as the result of others, in a local context, building on one another through interpretation. Bakhtin's (1981) influence is therefore evident in Schiffrin's work. She highlights the importance of text and context being interrelated in developing coherence across utterances. Schiffrin (1994) is particularly interested in the contextual information. She considers it is impossible to talk about something outside a contextual reference of some kind. Each person in the dialogue has to determine what the reference means and continue the dialogue according to their interpretation. In this way new interpretive contexts are continually developed. These ideas are interesting, and fit well with my own interest in the influence of contextual references in sharing understanding of meaning of mathematical words. The teacher is a 'discourse guide' helping children to move from 'everyday discourse' to 'educated discourse' (Mercer, 1995). 'Educated discourse' includes appropriate contextual references to exemplify meaning of specialist words. Children can also learn to explain their reasoning and share different ways of thinking mathematically, using specialist vocabulary.

Mercer (1996) identified three levels of analysing peer talk. The *linguistic* level is the types of 'speech acts' and exchanges that take place. It is about the content and the function of the talk, or *examining talk itself* (Pirie, 1997). The next level is a *psychological* one that analyses talk as thought and action. This includes the ground rules, the lines of reasoning, interests and concerns. In order to make judgements about the educational value of any talk Mercer (1996) suggested a third level of analysis, the *cultural* level. This clearly relates to the idea of an 'educational discourse', which in my research is the discourse of the mathematics classroom. Further development of these levels by Wegerif and Mercer, (1997) resulted in a modified interpretation that splits them into four and includes a focus on particular words. Linguistic research methods and levels of analysis therefore develop to suit a particular project. Mercer (1996) also categorised peer talk into three types, disputational, cumulative, and exploratory.

Disputational talk occurs when children challenge each other's ideas often through assertion and counter-assertion. Cumulative talk builds positively on the previous utterances, and exploratory talk offers statements for joint consideration, challenge and justification. Each of these types might be useful in analysing the dialogue during structured interventions. However, not all observed speech fits neatly into categories. One must also not lose sight of the fact that all discourse contributes to the meaning developed at a broader institutional level (Fairclough, 1995). Fairclough asserted that 'critical discourse analysis' includes consideration of institutional influences. In my small research project, clear identification of such influences is difficult, except to have awareness that the children come to the class having been educated into the discourse practices of the previous class, and the 'general' expectations within the institution. The power relationships are also important and not to be ignored, especially the influence of the teacher, or a more dominant peer in a paired situation.

Transcripts of dialogue taken alone will never present the whole picture (Edwards and Westgate 1994). Edwards and Westgate question the ability of an observer ever to fully know and understand the interactions they see.

"Since so much more is understood than is ever said, how is the observer to know what the participants are taking for granted about or reading into the interaction....."

(Edwards and Westgate 1994 p 16.)

One can only attempt to explain information that participants share openly at an intermental level, but not on that remaining as internal thought or 'inner speech' at an intramental level (Jarvis and Robinson, 1997). It is important to analyse the dialogue as it emerges, as well as the completed text in the transcript (Hicks, 1996). Hicks promoted a multi-layered form of interpretive analysis, based on Bakhtin's (1981) ideas that even individual thought entailed social activity. In order to discover such thoughts one has to observe the dialogue in action. Hicks (1996) described this method of enquiry as *contextual* because the whole situation is part of the interpretation

of small sections of dialogue producing 'thick descriptions' (Geertz, 1973 cited by Hicks). Another description is *micro ethnographic* because it explores 'moment to moment' constructions of meaning (Hicks 1996).

Although I intend to explore 'moment to moment' construction of meaning, my research is different from that of Hicks (1996) because I involve myself in dialogue with the children. The children are unused to explaining ideas to each other in the way I expect of them. My purpose in engaging in dialogue at appropriate times is to aid the development of dialogue between the children and thus enable me to observe the communicative processes involved. I must take into account the effects of my intervention. Consideration of all communicative events between the children and me are an essential part of my research.

### **The purpose of the pilot study**

The main purpose of the pilot study was to trial different methods of collecting data for analysis. During November 1997, the pilot consisted of one familiarisation visit, a pre-lesson discussion with the teacher, one lesson observation, a post-lesson interview, and a series of 'structured interventions' with nine pupils from the class, ten days after the lesson.

I observed one lesson with the purpose of exploring various means of collecting evidence of talk. During the videotaped whole-class teaching episodes, I made written observations, and recorded on audiotape, to compare the quality of information collected by each method. During group work, I used the video-recorder, an audiotape recorder with a tabletop microphone, a battery operated dictation recorder, and written narrative. I tested a different method with each group. My aim was to record child-child dialogue, teacher-child dialogue and researcher-child dialogue without being too intrusive. I also observed the teacher's and the children's reactions to the different methods, as well as considering my own preferred way of working. In the structured interventions, I provided the children with a specific mathematical word that they said aloud; drew a picture of; wrote a symbol for; and described in a sentence. The research model is based on Otterburn

and Nicholson's research (1976), with 300 secondary pupils as described in Dickson *et al.* (1984 1993 reprint). Appealing record sheets (Appendix 1) and a scoring system, provided motivation for primary pupils. Much of the evidence was on paper, although video captured the dialogue. I worked uninterrupted in the library area where space allowed a suitable distance for the video camera to capture all communicative exchanges. The initial exercise was followed by 'situations' cards that provided the word set in a pictorial and/or textual context, for which the children had to decide 'true or false' (Appendix 2). I chose to research was fractional language e.g. half, three quarters, one whole. On the teacher's advice, 'structured interventions' were organised into the children's usual mathematics lesson attainment groupings with three children in each mixed gender group. I agreed with him that the children might work more confidently in familiar groups, although I did not plan to compare responses from different attainment groups initially.

### **The impact of the pilot study on my research methods**

After experimenting, I concluded that video recording was the most effective mode of data collection. The audio sound, together with the visual images provided evidence of gesture, and other features to aid the analysis of dialogue. However, it was only suitable for the whole class teaching episodes and recording any dialogue during class group work, other than teacher-led dialogue, proved ineffective because of background noise. Even a high quality tape recorder with a tabletop microphone failed to collect recognisable dialogue. The hand held dictation recorder was, on occasions clear, but I had to be present to ensure its proximity to subjects. During group work, I found brief handwritten notes most useful, but I became aware of my limited focus of attention. While I noted incidents and dialogue that caught my attention, I must also have missed others.

Video recording during the pilot lesson observation helped to determine the most effective and least distracting position for the camera. During the pilot, I sat with the camera directly focused on the teacher, the board, and some of the children (Appendix 3). I positioned it where the teacher suggested, but

he was conscious of the camera, which sometimes appeared to affect his responses to children. During the final phase, I moved with the camera to the side of the room, causing less distraction to everyone, and with more children clearly visible. A plan of the classroom helped me identify children participating in the dialogue (Appendix 3). Moving to the side resulted in the whole-class dialogue becoming more relaxed. However, this might also be a result of the teacher's increased confidence by the summer term.

At the end of the pilot lesson observation, I quickly constructed a few questions, as a basis for a semi-structured, audiotaped interview. On later analysis I realised that my focus in the interview, although relevant, did not provide me with information that would support or refute ideas I had about the whole class dialogue after transcribing the videotape. In order to use the interview with the teacher as a triangulation, I decided to view all the final study lesson tapes before constructing questions, and to allow the teacher to watch the tapes as well, before answering the questions. I also decided to give the teacher a questionnaire, providing more time for completion, and I later spoke to him informally to clarify any ambiguities. I have included a discussion of these issues earlier in this chapter.

The first two groups of children in the pilot structured interventions responded positively to using the recording sheet (Appendix 1), but the middle-attainers had more difficulty determining the expectations so their verbal and written responses were less clear than the higher attaining group. I decided to abandon the use of this sheet with the lower attaining group and having a group discussion with them proved insightful into their communication processes. Working in this way demonstrated that the children were more likely to discuss, explain and challenge each other's way of thinking. However, my presence was too influential in either restricting or developing the children's thoughts, as I inevitably became the person in control i.e. the teacher substitute. This indicated that a mainly oral approach to encourage paired pupil-pupil discussion was most appropriate for the final phase. There was little evidence that children were affected by the

presence of the video camera once they became involved in the activity so I decided to continue to use it in the final study.

During the pilot, most instances of children entering educated discourse occurred during the structured interventions, and not during the classroom observations. An interesting event highlighted the importance of my intervention on occasions. Although I was not trying to determine levels of mathematical ability or understanding, I wanted to identify why one child could not explain the phrase 'mixed number' (Extract 3.1). By intervening, I was able to discover information that would not have become available if I had not intervened.

### Extract 3.1 Mixed number

Res. = researcher, Jake =child

Jake Mixed number

Res. Do you understand it?

Jake No

Res. You're not sure what a mixed number is. Have you ever heard of that word – mixed number?

Jake *(Shakes his head)*

Res. What do you think it might be?

Jake *(Looks thoughtful)*

Res. No idea at all?

Jake No *(quietly, shakes head)*

Res. If I give you some choices .....

*(Res. writes  $\frac{1}{4}$   $\frac{4}{1}$  and  $1 \frac{1}{4}$ )*

Res. Which of those do you think might be a mixed number?

Jake It's that one *(Points to the middle one)*

Res. Not quite...why have you chosen that one?

Jake I don't know.

Res. What would you call this one? *(Points to  $\frac{1}{4}$ )*

Jake A quarter

Res. Yeah, and what would you say that one was...where the four is at the top? *(Points)*

Jake	<i>(Looks but no response)</i>
Res.	Not sure?
Jake	<i>(Shakes head)</i>
Res.	Have you ever worked with numbers like this? <i>(Points to the mixed number)</i>
Jake	<i>(Nods)</i>
Res.	What did you do with them, when you had numbers like this?
Jake	Added them.

He explained the addition of  $1 \frac{1}{4}$  to  $3 \frac{3}{4}$  showing clear understanding that  $\frac{1}{4}$  plus  $\frac{3}{4}$  made one whole. Therefore, the child had conceptual knowledge without knowing the associated terminology. His knowledge was possibly the result of previous teaching that focused on symbolism and operations, as found in several previous British studies (Dickson *et al.* 1984; Hart, 1989; Kerslake, 1991). In addition to providing support for researcher intervention, the interaction also highlights the importance of providing opportunities to communicate using pencil and paper, and not through speech alone. Another important aspect was that we communicated through gesture and facial expression. Another child might not have responded to his communicative responses in the same way. As a researcher, I was deliberately trying to elicit information, while another child most likely would not.

The following chart (Figure 3.2) compares the two phases of my research process and identifies clearly the changes made mainly due to the pilot study experience. Some changes, such as collecting samples of work, arose during the final phase in response to observed events.

**Figure 3.2 Comparison of pilot and final phase methods**

<b>Pilot phase</b>	<b>Final phase</b>
<b>Strand 1 – Lesson observation</b>	<b>Strand 1 – Lesson observation</b>
<ul style="list-style-type: none"> <li>▪ Observation of one lesson</li> <li>▪ Use of narrative system, video recording and audio recording to compare the quality of data collected.</li> <li>▪ Teacher interviewed immediately after the lesson</li> <li>▪ Teacher not commenting on the tape of the lesson</li> </ul>	<ul style="list-style-type: none"> <li>▪ Observation of three lessons</li> <li>▪ Use of video recording for the main teaching activity</li> <li>▪ Occasional use of narrative system for teacher-pupil or pupil-pupil interaction during completion of class-work</li> <li>▪ Collect samples of written work</li> <li>▪ Teacher questionnaire designed after initial scrutiny of tapes from the series of lessons</li> <li>▪ Teacher completing a questionnaire on the tape material from the lesson</li> <li>▪ Informal discussion to clarify teachers written responses</li> </ul>
<b>Strand 2 – Structured intervention</b>	<b>Strand 2 – Structured intervention</b>
<ul style="list-style-type: none"> <li>▪ 9 children chosen by the teacher</li> <li>▪ Children in mixed gender attainment groups, of three; varied intervention from me.</li> <li>▪ The children's activity presented as a game with written recording</li> <li>▪ Children's activity provides written and video evidence.</li> <li>▪ Video records of all activity</li> </ul>	<ul style="list-style-type: none"> <li>▪ 16 children chosen by the teacher</li> <li>▪ Children working in mixed gender attainment pairs; my intervention as necessary</li> <li>▪ The children's activity presented as a series of sheets to encourage discussion</li> <li>▪ Children's activity provides written and video evidence.</li> <li>▪ Video records of all activity.</li> </ul>



### An outline of the final phase

The final phase occurred in May 1998 to facilitate research of the same class. Year 5 was the most appropriate year to choose, as research did not interfere with preparation for national testing. The children had already learned a range of mathematical vocabulary, and during the second half of Key Stage 2, children should be able to explain clearly their reasoning for a mathematical event or clarify their understanding of a mathematical word. Over a period of three consecutive days I observed and video recorded the daily mathematics lessons:

Lesson 1 - Revision of probabilistic language such as certain, likely, unlikely and impossible.

Lesson 2 – Introduction to the phrase ‘even chance’

Lesson 3 – Introduction to the probability word scale.

The lessons provided an opportunity to gain a sense of the processes involved when the teacher aims to provide continuity in the conceptual and linguistic development of the mathematical topic. After my initial scrutiny of video recordings, I passed them to the teacher with a questionnaire, during July 1998. The teacher returned his responses in September 1998.

Structured interventions, five days after the lessons, took three days to complete in a corridor by a stair well. There was some disruption on occasions, when children filed past, or when teachers opened their doors. During the pilot, the library was a much quieter environment. However, most children continued without distraction, and the video recorder picked up most of the dialogue. Each mixed gender pair, worked with me for approximately 30 minutes. Words chosen for discussion were: probability, even chance, certain, most likely. After the lesson observations I decided that the meaning of the phrase ‘even chance’ would be most interesting to explore with the children because the second lesson provided several interesting uses of contextual references. I chose ‘probability’ because the teacher had not explained its’ meaning explicitly and I wondered what meanings the children would give. ‘Certain’ was a word that many children only seemed to be able to use in the past tense e.g., “I am certain I ate my breakfast this morning,” and they had more difficulty using it as a



Figure 3.4 'Activity Sheet' instructions

Instructions – given orally and with demonstration.

The aim of the activity is to explain each mathematical term to your partner so that your partner can understand you.

- First, read the word aloud.
- Then explain what it means to your partner. You may draw as many different pictures as you like to help you explain. Your partner and I may ask questions.

Uncover the true or false statement

- Decide whether the statement is true or false
- Give reasons for your choice

The use of my *true or false* statements requires justification. The idea of being able to make a judgement on whether a statement is *true* or *false* relies on everyone reading the statement to have the same understanding of it. The statements used may not represent a mathematician's view of 'precision' in relation to mathematical concepts. The statements represent something a child might say, or hear an adult saying in everyday use. I constructed the sentences with no effort to make them correct in a mathematical sense. If they are not mathematically correct or incorrect then perhaps the idea of asking children to make a judgement of truth or falsehood is questionable. I am using **true** as 'in accordance with reality or reason' and **false** as 'misrepresenting reality'. When producing the 'true or false' statements I found exemplar material in books. For example:

"There's just as much chance of throwing a five as there is a six"

(Brading and Selinger, 1993 p9)

Brading and Selinger provide this as a situation when there is likely to be an even chance of events happening. 'Even chance' suggests a choice of two

outcomes, so there is a 50% chance of an event being successful or a probability of  $\frac{1}{2}$  on a probability scale. The statement I used is:

“There is an even chance for each team to win the match”

The statement is a modification of Brading and Selinger's model. An ‘even chance for each team’ suggests a comparison of each team's chances i.e. the outcome will be that either team A or team B will win. This does not accurately reflect reality because there is always the chance of a draw. Some children may recognise this and also discuss other issues that affect outcomes e.g. each team plays on the same pitch, has the same number of players etc. My aim was to provide a provocative statement set in a ‘real-life’ context for discussion to explore the children's ways of explaining their understanding of the meaning of mathematical words. Whether they make a sound judgement of ‘true’ or ‘false’ is of less importance to me than the nature of their reasoning.

Structured interventions aimed to explore what the children thought with minimal intervention from me. The information gathered may support or contradict preliminary hypotheses about the use of contextual references drawn from lesson observation. I presented each consecutive pair with the same set of words in a different order each time. The purpose was to prevent children anticipating and preparing for, the order of my enquiries, and to minimise effects of ‘shyness’ or fatigue. The first word might not produce as much discussion because of the unusual situation and the last word because children are tired. The teacher also asked the children not to talk to their peers about what they had been doing until I had finished my work. The children took turns to be leader, and put their name on the paper. Theoretically, this meant that each child described two words, but sometimes both children became fully involved in dialogue and therefore fully engaged in making sense of all four words. This was the aim. I noted ‘even chance’ as a phrase to research because it evoked a range of language and contextual references from the children during the lesson.

In total, the final phase video material consisted of 2 ½ hours' lessons and 6 hours of structured interventions. To keep the transcript format simple, and to focus on most relevant features for later analysis the following symbols and conventions were used:

[	two speaking at the same time
...	pause of one second or less
.....	pause of more than one second
<b>bold</b>	louder, speech that is more emphatic
<i>italic</i>	quieter, more tentative speech
(inaudible)	unclear
(day)	possibly this word

There is no symbol for changes in tone, but where relevant, such changes are noted. My aim was to keep the system simple but effective for its purpose. My observations include contextual aspects considered relevant to the dialogue. For example, if the teacher asks a question it might be relevant to note the time given to the children to answer it before repeating or rephrasing. Tone of voice, gesture, and body language are all contextual aspects that might influence the nature of the dialogue. Alternatively, the dialogue might influence these. As the focus is on the effect of contextual references, and the way that participants share their understanding of the meaning of mathematical words, then the other aspects are only relevant if they affect this process. Therefore, in discourse analysis judgements are made and justified, depending on the influence of the aspect.

### Summary of Chapter 3

In such a small-scale study, it is impossible state whether a particular interactional style in the classroom is more likely to teach children particular mathematical words because the quantity of data is insufficient to draw conclusions. However, the dialogue can provide some clues as to which might be the most efficient or effective methods and of helping children learn particular words and provide the basis for theoretical discussion. My purpose is to explore the overall structure of the discourse and to determine how the children develop shared a understanding of meaning of

mathematical words and phrases. To this end, it is important to look at the social context of the situation, and specific contextual references which the teacher chooses to teach the mathematical words. Qualitative methods with a focus on discourse analysis are therefore most fitting for this project. My methods suit my research focus. I used the pilot project to develop and refine them. Two strands, the lessons and the structured interventions, merge to provide evidence of the ways in which participants share their understanding of the meaning of mathematical words through contextual references.

## **Chapter 4: Analysis**

### **Introduction**

This chapter is organised into three further sections and a brief summary. First, I provide a brief overview of findings from the pilot study in which I observed and collected data about fractional language. These findings had an impact on my initial hypothesis that contextual references most likely arise from the speaker's 'imagined' situational context that they perceive the word to describe. I explain the impact of the pilot findings and modify my working hypothesis about the nature and impact of contextual references on developing a joint understanding of meaning. Secondly, I analyse the final study lesson observations, on probability, in order to answer my first research question "*How does the teacher interact with the children to develop a shared understanding of the meaning of mathematical words?*" Each of the three lessons provides a slightly different set of data about the teacher's use of contextual references in order to provide opportunity for developing a joint understanding of meaning. The three lessons comprise a 'long conversation' about probabilistic words and phrases that allowed me to look at the historical effects of previous lessons. My third section analyses the processes by which the children share their understanding of meaning of the phrase 'even chance'. This section uses data that answers my second research question, "*How do children express their understanding of the meaning of specific mathematical words and phrases?*" In particular, I am interested in events that relate clearly to the observed classroom events where the contextual references have influenced the development of meaning. I analyse the structured interventions, and discuss the children's communicative events in two sections. First, I consider the effects of non-verbal modes of communication, gesture and recording on paper, alongside the discourse they accompany. Second, I analyse the effects of contextual references that influence children's reasoning processes. These are 'everyday' or pseudo real-life, and others that I consider to arise from the mathematical content of the lesson. For each research question, I outline key points that provide a basis for the summary in Chapter 5.

### **Refining initial ideas through the pilot study findings**

During the pilot lesson, the teacher revised fractional words and phrases by using them and by asking the children questions to demonstrate their knowledge. His approach encouraged a view of mathematics as a procedure (Cobb, *et al.* 1992). The teacher drew representations of fractions as parts of a region or collection and the focus was on the procedural knowledge of deciding how many parts had been ‘eaten’ or shaded. The teacher clearly demonstrated the relationship between the ‘everyday’ contextual reference to pizza and various shapes with associated fractional language and the symbolic written forms of fractions. There was no evidence, in the lesson, of developing a ‘shared understanding of meaning’ (Edwards and Mercer, 1987) of such words through use of dialogue and a range of contexts. Most teacher questions required short factual answers encouraging the children to *provide the answer they thought the teacher wanted*. There was little evidence of children using mathematical language except as occasional short factual answers. The set of specific contextual references, quite likely prevented the children developing a generalised idea of ‘fraction’. For example, the teacher used the word ‘whole’ with reference to pizza, or a ‘whole collection’ but not ‘whole numbers’.

During the structured interventions, some children had great difficulty conceptualising the number one as a ‘whole’ and subsequently a fraction as a part of one. The children mostly referred to the same contexts as the teacher, but when I introduced the idea of pocket money, the majority of children were able to discuss fractional parts as equal shares. This was obviously a very relevant context for them, particularly for the lower attainers who had great difficulty explaining and discussing the ideas associated with numerical fractions. Generally, the lower attainers engaged in mathematical talk that was meaningful to them, displaying awareness of some relationships using ordinary language (Pirie and Schwarzenberger, 1988). Observations during the structured interventions thus led to a new consideration of the importance of contextual references and how they might affect the shared understanding of meaning. When the teacher spoke of a whole pizza or whole set of objects and split these into their named



fractional parts e.g. two halves, he had the understanding that these are equivalent to 'one' in a numerical sense, the numerator in  $\frac{1}{2}$  representing one whole divided by the denominator, or the number of parts. Some children failed to make this connection, suggesting that the contextual reference limited their understanding of the meaning and so a joint understanding did not occur. This particular confusion can arise because numerically expressed a 'half' might also mean 'one of two equal parts'. If the children have this understanding of the meaning of 'half' then 'one' cannot be a 'whole'. Everything depends on how the children perceive the contextual reference. The children can either see the pizza as 'one whole divided by two', or one piece of the pizza as 'one of two equal pieces'.

Thus, as outlined in Chapter 2 my hypothetical idea about the nature of contextual references developed into consideration of how the speaker's 'image' becomes part of the thought processes of the recipient. Developing a joint understanding of meaning most likely requires the development of similar 'images'. The use of the pictorial image of a pizza, and asking the children to imagine eating the parts of the pizza, is a common approach to teaching fractions at primary school. However, it seems evident from my observations that a teacher cannot assume that the children develop the same understanding of meanings, even when clear links are made between the everyday context, the mathematical numerical context and the associated language. My hypothesis retains the idea of the spoken contextual reference arising from an imagined situation, but in teaching, this imagined situation often becomes a drawing or a physical model of the situation. The purpose of these teaching aids is to help the children understand the meaning of the spoken word by providing them with an opportunity to develop an imaginary situation of their own. It is not just a simple case of the children misunderstanding, but one of the children having a different perception of the contextual reference and therefore not developing a shared understanding of meaning with the teacher.

### Analysis of the lesson observations

Analysis of lesson transcripts identifies particular discourse practices used, and provides opportunity to consider the effectiveness of different strategies in the development of a shared understanding of meaning. To aid referencing I have numbered the short extracts of dialogue as figures, and each utterance has a number that identifies its position in the particular extract and in the longer section of transcript evidence from which it came. In my analysis, I refer to occasional utterances by number in order to set them into the context of the whole extract. I have changed children's names in order to retain anonymity. May I also remind readers of the transcript conventions described in Chapter 3:

[	two speaking at the same time
' ...	pause of one second or less
.....	pause of more than one second
<b>bold</b>	louder, speech that is more emphatic
<i>italic</i>	quieter, more tentative speech
(inaudible)	unclear
(day)	possibly this word

There is no symbol for changes in tone, but where relevant, such changes are noted. I have also modified the presentation of transcript extracts for the analysis to include observations in brackets {observations} where appropriate, rather than in a separate column. I believe such a change necessary for ease of referencing and making closer links between dialogue and events for the reader.

Brief discussion of the teacher's plans provided information that the aim of the first lesson was to revise the words used to describe probabilities. The second lesson focus was the phrase 'even chance' and the third lesson was a review of the language covered during the previous two lessons, and introduced the probability scale. The lesson objectives showed that the teacher expected the children to learn the words he introduced, but it was unclear whether he expected them to develop a technical discourse (Rowland, 1995). The teacher thought that the best way to secure an understanding of mathematical language was to use real life situations, and

to ask the children to explain in mathematical sentences (Appendix 5, Q8). However, children’s written phrases and sentences rarely included the mathematical words, although they did provide explanations for their answers.

In the first lesson, two identifiable types of reformulation had direct relevance to the contextual references introduced into the dialogue. References to particular transcript extracts are in brackets, for examples of these particular reformulations. These were:

**Modifying** – applying to a more generic contextual reference, but not generalising (Extract 4.2)

**Refining** – applying to a more specific contextual reference (Extract 4.3)

Another influence, generally during elicitation was:

**Generalising** – changing a specific contextual reference into one that represents ‘all’. (Extract 4.6, utterance 31)

Other categories used were a variation of knowledge markers used by teachers described by Mercer (1995), **agreeing** i.e. accepting the response e.g. “Good.”; “Well done.”; “That’s right.” (Extract 4.4, utterance 13). However, both the teacher and the children demonstrate use of, **challenging** e.g. questioning the wisdom of the speaker’s ideas during the lesson (Extract 4.4). Challenges from the children often resulted in an exchange that I describe as **agenda setting** i.e. organisational, maintaining order, or keeping the focus on the lesson objective (Extract 4.5)

An initial real-life context of the lottery gained the children’s attention and provided a focus for the introduction of the words ‘certain’, ‘might happen’, and ‘impossible’, presented in table format (Figure 4.1).

**Figure 4.1 Tabulating probabilities**

Certain	Might happen	Impossible
Some sort of injury	Win the lottery	For us to fly

The tabular image of three distinct categories is not representative of the tentative nature of real-life probability, in relation to the image perhaps becoming a mediator for thought (Kieren, 1993). After deciding that 'win the lottery' fitted in 'might happen', the teacher asked the children to provide their own examples of events to put in the table. In this way, there was opportunity for the children to make individual sense of the words within their own contextual framework. The children drew on their personal experiences that did not necessarily resonate with other children's experience. The teacher later commented that he tried to provide explanations for the other children in ways identifiable as 'building' the topic through interaction (Jarvis and Robinson, 1997). The teacher's responses, in particular the reformulations, varied according to the contextual reference introduced by the child. Each example demonstrates the child using the word as a label, in an attempt to categorise an event (Mason, 1999). The language was mainly presentational in nature (Barnes, 1992). Children rarely offered an idea for discussion, although the teacher did alter the nature of the references they used. First, a child referred to an everyday context as an event that is certain (Extract 4.2).

#### **Extract 4.2 Modifying**

- 2        Stuart: It's 'certain' that you are going to bang your head, *or hurt yourself or something like that....*
- 3        Teacher: It's 'certain' that you are going to have some sort of injury throughout your life. Yes?

The child provided a specific example then began to make it less specific, and the teacher continued to **modify** the relatively specific nature of the injury to a generic *some sort of injury* instead of a specific *bang your head*. However, there was an assumption that *some sort of injury* is certain, and that everyone understands the reference. The teacher's tone of voice and facial expression when he said "Yes?" to the class implied that he expected agreement. Although the exchange was clearly dialogic in nature, with the teacher building on the child's utterance, it was short. Perhaps the "Yes?" coupled with him writing on the board, also signalled to the children 'time

to move on'. The teacher was setting expectations for the discourse, and defining the language as he perceived it in order to establish meaning (Barnes and Sheeran, 1992). The dialogue continued with the teacher quickly responding to another child (Extract 4.3).

#### Extract 4.3 Refining

- 4      Colin: It's 'impossible' to fly  
 5      Teacher: It's 'impossible' to fly as in ... {Teacher gestures - bird flapping it's wings (to Colin)}  
 6      Colin: Wings.  
 7      Teacher: Yes.  
 8      Child: (inaudible) Fly in a 'plane.  
 9      Teacher: I know we have 'planes but generally it would be 'impossible' for us to take-off ourselves *without a 'plane...*

Here the teacher **refined** Colin's response with the aid of a deliberate gesture to make it a more **specific** reference to flying like a bird. The teacher changed the child's contextual reference into one he considered a better match to the term used, in an attempt to ensure joint understanding. Gesture is useful to clarify the meaning of words (McNeill, 1985). The teacher used gesture instead of words to elicit a response from Colin. Another child **challenged** the teacher's assertion that we cannot fly (8), and suggested a 'plane. The teacher seemed unprepared for the challenge and determined to keep the focus, so he **modified** the response (9). Circular gestures accompanied the response that did not exemplify meaning, but perhaps demonstrated the teacher's difficulty in finding words to respond. Questionnaire responses identified that the teacher was unprepared for the ambiguity created by working with 'real life' contexts (Appendix 5, Q5) as children continued to challenge the references made (Extract 4.4).

#### Extract 4.4 Children challenging

- 12      Nita: It's 'certain' that we are going to have one birthday every year.  
 13      Teacher: Good, it's 'certain' that we will have one birthday...

- 14 Child 1: No it's not because you might die
- 15 Teacher: ...Every year...
- One** birthday.
- 16 Child 2: I'll have (inaudible)
- 17 Child 3: But you could be born in a leap year
- 18 Teacher: A leap year, yes, but ... shhh! They usually celebrate every year, but I see what you mean, it could be a 'might happen' if it's a leap year...

The teacher **agreed** with Nita's idea by 'confirming' and 'repeating' the response. These are 'knowledge markers' that teachers use to identify knowledge as significant and joint (Mercer (1995). Other children **challenged** the labels given to events, and provided arguments to dispute the original assertions. This is more characteristic of disputational peer talk (Mercer, 1996). They referred to other contexts that helped them challenge the original idea. The teacher ignored the first challenge, and raised his voice to maintain the focus, but then he **agreed** to the idea of a leap year, and elaborated for the rest of the class.

In the next section (Extract 4.5), the teacher **set the agenda** by tone of voice and a change of focus to the prediction 'impossible'. However, the children had great difficulty providing examples for 'impossible' events and Jake and Kit's suggestions (27,29) appeared silly to the teacher (30). The teacher **challenged** the children, not always giving reasons why they were not correct. After watching the tapes, he questioned whether the children's intention was to be silly (Appendix 5 Q10). He recognised that the children perceived the use of the words from a personal viewpoint rather than one that would include everyone. This seems to match with research into children's understanding of probability, that shows children basing their earliest probabilistic thinking on subjective judgements (Jones *et al.* 1997). The teacher's decision not to continue with the 'metacognitive' idea that Julie raised (23,25) reflects his difficulty in maintaining a coherent and comprehensible discussion with the class. He was confused about her

meaning, and informal discussion with him confirmed that he considered it too complex for full class debate.

#### **Extract 4.5 Agenda setting**

- 18cont. ...some more 'impossible' ones. Matt?
- 19 Matt: I know one 'might happen'...
- 20 Teacher: 'impossible' please first, alright? Sophie?
- 21 Sophie: A five year old would get in to see Titanic.
- 22 Teacher: Yes.....that a five year old would get in to see Titanic...Julie.
- 23 Julie: It's 'impossible' to erm...you know, think back (wards) at (inaudible) you know if you've (inaudible)... to think of that...to think it
- 24 Teacher: Yeah, but it's not 'impossible' to think as we are thinking now.
- 25 Julie: Yeah, but thinking of something now as it hasn't happened...
- 26 Teacher: We'll leave that one now as I'm a bit confused about that one. Jake.
- 27 Jake: It's 'impossible' not to watch Coronation Street.
- 28 Teacher: It's **not** 'impossible', Kit.
- 29 Kit: It's 'impossible' not to eat McDonalds.
- 30 Teacher: No that's not 'impossible' either...erm... **bit silly now...** I want ... I thought... Year 5 would have been a more sensible. Actually, I'm a bit disappointed...

It seemed that the teacher was expecting particular types of answers. From the lessons in the pilot, and the teacher's comments there is evidence that he usually guides children to the answers he expects in order to provide a model for other children (Appendix 5 Q6). This is a common occurrence in British classrooms (Gardiner, 1992). Steinbring (1991) also comments on teachers structuring lessons on probability in such a way that children fail to understand the dynamic nature of the concepts. It seemed that in this particular lesson there was a danger of this occurring.

During group work, the teacher focused children's attention on a 'normal' wood (Extract 4.6, 31) and attempted to '**generalise**' the context of Atholl Wood presented in the textbook. This caused various responses from the children demonstrating the teacher's difficulties when children have a different perception than his own. He led the children into the answer he was looking for, but some children challenged his **agreement** that it was verging on 'impossible' to see a tiger in a wood. A focus on ensuring the children understood the given pseudo-real life context of Atholl Wood, as described in the textbook might have avoided such problems.

#### **Extract 4.6 Using the textbook**

- 31 Teacher: **Number seven**, think of a normal wood in this country.  
Not a zoo, alright? "They will see a tiger". They will see a tiger.  
Stuart
- 32 Stuart: 'Very unlikely'.
- 33 Teacher: 'Very unlikely', verging on which one really? Lee?
- 34 Lee: 'Impossible'. {A number of children speak}
- 35 Teacher: 'Impossible'... alright? Shh! I have just ... What?
- 36 Child: It might have escaped.
- 37 Teacher: 'Very unlikely', what do you say Mrs Bold?
- 38 Mrs Bold: 'Very unlikely'.
- 39 Child: It might... {other children speaking too}
- 40 Teacher: 'Very unlikely'. **Number eight**, "They will find feathers".

The teacher's use of 'very unlikely' might be confusing as he argued that 'see a tiger' was almost 'impossible', while the children placed it at 'very unlikely'. The teacher's appeal to me for support is indicative of the pressure he felt in controlling the dialogue while allowing the children to present their own ideas. The children might be prepared to consider the 'unlikely' event as degree 'more likely' than the teacher does, just as they more freely enter into imaginary play than an adult does. Earlier in the lesson the notion of 'flying to the moon' prompted differing views and it seems that children are more able to imagine the possibility of such an event than an adult. They make sense of the whole situation, as they perceive it



(Confrey, 1999). When we say 'very unlikely', we can mean 'more unlikely than unlikely' or 'almost impossible' it depends on where we perceive the situation to belong on the continuum. One difficulty might also be that the teacher had not presented the children with the visual image of a probability scale, and so it was confusing to the children to talk about 'verging on impossible'. Another consideration is that children might remember an actual news story about an escaped tiger, thus making such an event 'possible', but still 'very unlikely', rather than 'impossible'. It is perhaps easier to identify an event as 'very unlikely', than 'impossible' in a similar way to children finding 'possible' events easier to identify than 'certain' events as identified by Fischbein *et al.* (1991).

Later, the textbook focused on numerical quantities for considering likelihood and this type of reference seemed much easier for children and the teacher to relate to probabilistic words. Referring to numerical quantities made justifications clear, allowing the teacher's **agreement** without reformulation. The teacher also considered that using the language 'most likely' and 'least likely' was easier for the children (Appendix 5 Q9). The teacher's observation supports Fischbein's *et al.* (1991) research that found children had difficulty with events that were 'certain'. The act of comparing two numerical quantities is much easier than making judgements about events without numerical quantities attached. Piaget and Inhelder (1975) used experiments with numerical outcomes, as did Jones *et al.* (1997) in their research. During such research, stages of conceptual development in determining probabilities are easier to identify because the variables affecting likelihood are clearer than in 'everyday' events. Children are more able to comment on the causes of a particular event with fewer variables. Teaching probabilistic language through reference to everyday events possibly hinders the development of probabilistic reasoning in terms of cause and effect, and therefore hinders the process of developing a shared understanding of meaning of probabilistic language. Fischbein *et al.* (1991) also considered the use of everyday contexts unhelpful for teaching probabilistic concepts. However, the teacher thought that real-life contexts were the best approach (Appendix 5 Q8), but admits that less ambiguous

contexts would be better (Appendix 5 Q5). Having numerical quantities attached to the context reduces the ambiguity e.g., when 20 beads in a box are all white, you are 'certain' to pick out a white one.

During the second lesson, pupils introduced further contextual references when trying to share their understanding of the meaning of 'even chance' (Extract 4.7)

**Extract 4.7 'Even chance'**

- 1 Teacher: What do you think 'even chance' might mean? Think about what the word 'even' means, and the word 'chance'. Tim?
- 2 Tim: Well, say (...old) you've got an 'even chance' of living and you've got a chance, you've got half a chance that you'll die and you've got half a chance that you'll live.
- 3 Teacher: Super. Half a ch.... Tim has given the scenario of living and he says you've got an 'even chance' of living, which also means you've got an 'even chance' of not living. Right he's used half-and-half...Anything else? Jake?

The teacher began by asking the children the meaning of 'even chance'. He had a clear focus on teaching the meaning because of the difficulties encountered in Lesson 1 with children giving him subjective and intuitive responses. However, the teacher's cues could have been a source of confusion for some children. 'Even' could mean the same; smooth; equal; flat; balanced; divisible by two, or any variation of these. Tim's descriptions of 'half a chance' suggest that Tim was thinking of two equally likely outcomes in a real-life context. A single event with two equally likely outcomes has a probability expressed as half on a probability scale (DfEE, 2001). However, Tim's notion of having 'half a chance' of living or 'half a chance' of dying is not simple in real life. Doctors' medical knowledge helps them make informed judgements of a patient's chances based on available statistical data. It is rarely 'even'. However, from a child's viewpoint a person can 'dead or alive' in a similar way to 'heads or tails' on the toss of a coin. The teacher **refined** the child's contribution by

reintroducing the term ‘even chance’ and used ‘not living’ rather than ‘dying’. Informal discussion revealed the teacher thought ‘not living’ a better option. The children seemed unperturbed by the different ways of referring to life and death. The teacher focused on ‘half and half’ as another label for ‘even chance’, without dialogue about the more complex issues described above. His purpose was to maintain the focus on the new term ‘even chance’ and to prevent recurrence of the previous lesson’s confusion. He taught the children that different labels exist for the same mathematical situation but did not necessarily develop a shared understanding of meaning with the children. Jake’s contribution introduced another label for ‘even chance’, ‘fifty-fifty’ (Extract 4.8).

#### **Extract 4.8 Fifty-fifty**

- 4      Jake: The tackle was ‘fifty-fifty’.
- 5      Teacher: The tackle was fifty-fifty. Explain that. There’s non-  
             footballers in the class.
- 6      Jake: It means that you’ve made a challenge.
- 7      Teacher: An even challenge, yep. Anyone else? No,...Right!

The teacher **agreed**, requested an explanation then specified the context, football thus **refining** the response. Jake described ‘fifty-fifty’ as a challenge and again this was **refined** to ‘even challenge’. It seems a clear attempt to ‘build’ on the children’s ideas of ‘even chance’ following a similar whole class lesson structure to that described by Jarvis and Robinson (1997). Making sense of a commonly used expression ‘fifty-fifty’, to describe an ‘even challenge’ in a tackle, relies upon making sense of ‘even challenge’ in this extract. Probability considers possible outcomes. If ‘even challenge’ suggests there is an ‘even chance’ a player will gain or lose the ball, then outcomes are the focus. However, the children might be thinking in terms of two players being an ‘even’ distance from the ball, or running at an ‘even’ speed towards it. The difficulties in referring to ‘real life’ examples are evident when considering joint understanding within the whole class again supporting findings by Fischbein *et al.* (1991).

Later experimental work with repeated trials included tossing coins and cups, and choosing coloured counters, to clarify the meaning of ‘even chance’ numerically. When the likelihood of events is unambiguous i.e. tossing a coin can only have two equally likely outcomes, heads or tails, then pupils learn the meaning in a relatively precise contextual reference. However, the children became involved with the procedures of the ‘experiments’ rather than the prediction of the outcomes. Perhaps without realising the teacher set up another situation that encouraged a focus on procedural rather than conceptual understanding (Cobb *et al.* 1992). It is interesting to note that the teacher moved from asking questions that invited ideas in the introduction to the lesson, to questions that required specific responses as part of his strategy to reduce ambiguity and ensure joint understanding of meaning for the main part of the lesson (Extract 4.9).

#### **Extract 4.9 Experiments**

7cont Teacher: I've got some cubes, alright? I'm going to put them in...it's a little memory game. You've got to remember three red cubes and two blue cubes. How many red Naomi?

8 Naomi: Three

9 Teacher: How many blue David?

10 David: Two.

11 Teacher: Right, do you think...hands up if you think if I said, 'pick a cube', it would be '*even chance*'... {No hands up}. Hands up if you think it would be '*uneven chance*'... {All hands up}

**OK**

Richie, explain why.

12 Richie: Because there's more red than blue.

The situation was simple, with a small number of cubes. All the children responded accurately when asked to put their hands up (11). However, the teacher introduced a non-standard phrase ‘uneven chance’ in an effort to consider what the opposite of ‘even chance’ might be. It might be that the teacher had difficulty explaining, so used an everyday word to simplify the situation. The teacher later questioned his own wisdom in using it

(Appendix 6 Q1). If an event's chance is not 'even' then its position is somewhere else along the probability scale. The children did not question it presumably because they were familiar with the word 'uneven' in everyday contexts and because they had not yet positioned 'even chance' on a probability scale.

One activity involved a 1-6 die, and the teacher **agreed** with a child's response (Extract 4.10 below). Stuart's suggestion implied that having one of each number on a die meant 'even chance'. The agreement has the potential for misinterpretation of the meaning of 'even chance' when a die is involved. However, the teacher refined the original question by asking the children to think of two outcomes, thus eliciting the correct response. The teacher **agreed** and elaborated so that the class understood the task (21).

#### **Extract 4.10 'Even chance' with a dice**

- 17 Teacher: ... First one..... It's... with a dice... alright? How could use the dice to show 'even chance'? How could the shake of a dice show 'even chance'? Stuart.
- 18 Stuart: It's like there's one of each number.
- 19 Teacher: There is one of each number right. How else could we do it is so that there is only two outcomes? Before we had; I know it is difficult to think; there was two outcomes. Nita?
- 20 Nita: Even and odd.
- 21 Teacher: Even and odd. That's what we're going to do, alright? We are going to see if when we shake the dice it's an even number or an odd number.

By agreeing with Stuart, the teacher might have reinforced an incorrect view that the possibility of throwing any number on a die is 'even chance'. It is most likely that the teacher was trying to keep his response in a positive framework. The outcomes are 'equally likely' but there is a 'one in six' chance of getting a particular number, not 'one in two'. Incidents such as this highlight the difficulty in teaching though a whole-class interactive teaching approach as recommended by the NNS (DfEE, 1999a). The teacher

is encouraged to use a variety of communication strategies such as explaining, modelling, and questioning to work with the whole class. The teacher has to think quickly, and respond positively while at the same time ensuring development towards a clear understanding of meaning for each member of the class. It seems inevitable that some elements of the communication process will be ineffective, as different children attach a different meaning to each utterance.

During Lesson 1, several children **challenged** ideas and the teacher did not usually accept the challenges as valid. He found the lesson difficult and felt that clear-cut examples would be better (Appendix 5,Q5). In Lesson 2, there is evidence of the teacher accepting a child's challenge more readily. He built on Sophie's challenge (Extract 4.11, utterance 35) to try to make sense of 'even chance'. Perhaps this was an indication that he was more at ease with the flow of dialogue in this lesson. It is here that I noticed small, possibly sub-conscious gestures, associated with the phrase 'even chance' (36) that appeared to support the spoken words in helping the recipient understand the meaning in the way described by McNeill (1985). His gesture to exemplify 'even chance' was his hand moving equally from side to side as if smoothing a surface with a balanced motion.

#### **Extract 4.11 Gesturing 'even chance'**

- 31     Stuart: It's got to be same chance.
- 32     Teacher: What does it make it though? Same chance.
- 33     Stuart: Fair.
- 34     Teacher: Fair .....{looks at Stuart}test. A fair test.  
          Alright?...Sophie?
- 35     Sophie: Sir, it won't be a fair test because the open end will come  
          down more easily.
- 36     Teacher: No, that doesn't mean it won't be a fair test. That's  
          depending on whether it's an 'even chance' or not. OK. {with a side  
          to side hand gesture}

Again, there was opportunity for confusion of meanings as Stuart and the teacher both identified the 'same chance' as a 'fair test'. The teacher attempted to elicit the expected response from Stuart by eye contact and waited approximately 3 seconds, but then provided the answer himself. Sophie **challenged** the notion of the 'fair test' based on her hypothesis about the outcome.

In the plenary, the teacher led the children to the outcomes and language he expected from one experimental situation as he gathered the results together (Extract 4. 12).

#### **Extract 4.12 Reinforcing labels**

- 56     Liam: We got ten red and ten blue.
- 57     Teacher: Ten red...What can you say about that then  
Liam?...mathematically.
- 58     Liam: 'Even chance'.
- 59     Teacher: 'Even chance'...what else could you say? Jake.
- 60     Jake: Fair test
- 61     Teacher: Yeah, I was thinking of something else. Sophie?
- 62     Sophie: 'Fifty- fifty'.
- 63     Teacher: 'Fifty-fifty', I was thinking {small rocking gesture with  
one hand}...you got what you expected didn't you?...

The teacher elicited the terms he wanted from the children, 'even chance' and fifty-fifty. A small, balanced rocking gesture exemplified 'fifty-fifty', which was different from the previous side-to-side balanced gesture for 'even chance'. This difference is interesting because it suggests that fifty-fifty is like a rocker balance, while even chance is like a smooth, flat beam. However, the two different phrases have the same mathematical meaning. If the gestures are an important part of sharing the meaning, then the difference might cause children to consider the two phrases as having a different meaning. The teacher's aim was to link the language 'even chance' with an equal numerical outcome. However, Jake introduced 'fair test' again and the teacher **agreed**, thus causing another opportunity for confusion.

Subsequently results of five groups were collated and totalled reinforcing the idea of 'fifty-fifty' with the visual image of 100 square on the board and a child drawing where the expected line would be to show 'even chance' from one hundred trials. The teacher thought the children successfully learned 'even chance', because of the clear focus in the lesson (Appendix 6, Q 2 and Q3).

The teacher introduced the probability scale during lesson 3 while revising probabilistic language. The children identified it as a 'time-line' from a similar activity in a history lesson. The mid point of a 'time-line' can represent any time, depending on the value of the limits while that on a probability scale always represents  $\frac{1}{2}$ . As the children did not work with a numerical scale, the analogy did not seem to cause problems during the lesson. However, although the development of images is important (Pendlington, 1999) it is possible that some children developed a perceptual meaning with regard to the placement of the events on the probability scale that was conceptually inaccurate. However, the teacher was positive about the response to the scale (Appendix 7 Q2). Most interesting is the dialogue referring to the event 'see a rabbit' (Extract 4.13, 1) which is placed at the mid-point on the scale.

#### **Extract 4.13 The probability scale**

- 1 Teacher: 'Very unlikely'. "See a rabbit" ... how would you describe that? What words would you use to describe where that arrow is?  
Ellie?
- 2 Ellie: Is it 'unlikely'?
- 3 Teacher: Pardon?
- 4 Ellie: Is it 'likely'?
- 5 Teacher: Is it 'likely'? Sally?
- 6 Sally: Between 'unlikely' and 'likely'.
- 7 Teacher: Between 'unlikely' and 'likely'. Any other words we could say then, to describe that, Stuart?
- 8 Stuart: Between 'likely' and 'unlikely'.
- 9 Teacher: Between 'likely' and 'unlikely', in the middle of...Chris?



{Chris gestures a rocking motion with one hand and the teacher copies}

...That's not saying that much! Susie?

10 Susie: It'd be both.

11 Teacher: Yeah, it could be. That's a good one actually; it could mean that it could be either 'unlikely' or 'likely'. Maybe depending on the time of year and where they were. Liam?

12 Liam: Quite 'likely'.

13 Teacher: Quite 'likely'. Sophie is your hand up?

14 Sophie: Fairly 'likely'.

15 Teacher: Fairly 'likely'. Go on.

16 Child: May be

17 Teacher: May be. Richie?

18 Richie: 'even chance'.

19 Teacher: Super Richie, 'even chance' ... 'Fifty-fifty' chance...it's sort of...it's in the middle of the line but it's between the two that are inside of it isn't it? Right, that's good Richie well done lad.

Ellie changed her answer after the teacher said, "Pardon?" perhaps thinking that her first answer must have been incorrect. The teacher repeated her tentative response, but then sought further answers. Much of the dialogue consisted of the teacher repeating responses, as if **agreeing** followed by further elicitations until eventually Richie gave the required response (18). One might question the notion of 'even chance' in relation to 'seeing a rabbit' and ask the children whether they thought this was a good prediction of the outcome of a walk in Atholl Wood. The children need to develop their skills of reasoning, and thinking about causality of events if they are to really understand the meaning of 'even chance'. The main problem with 'see a rabbit' is that it would be almost impossible to make such a prediction unless one had some data recording that people saw a rabbit on 50% of their visits to Atholl wood. Other contextual references introduced by the textbook and by the children, as 'even chance' events were also difficult to justify, thus creating a need to provide numerical data from which to make predictions for real-life events. In my experience, the curriculum material in

primary schools rarely supports the idea that probability is part of the process of handling data by linking the topic with the use of numerical data for predicting outcomes of real life events. In fact, this link only becomes explicit in Year 7 of the NNS Framework (DfEE, 2001).

The teacher reinforced the two labels identifying the half way position on the probability scale (19). The rocking motion with the hand used by Chris perhaps indicated that he could not think of the words but remembered the teacher's gesture from lesson 2 as discussed previously. Considering that lesson 2 focused on 'even chance' the children had great difficulty transferring the meaning they developed for 'even chance' from the lesson, to meaning it had on the on the probability scale. In lesson 2, equal numerical outcomes meant 'even chance'. Such a model is very different from placing 'even chance' along a continuum. The way 'even chance' was represented in Lesson 2 might have limited transferability of its meaning to a new representation, in a similar way to the limitations of transferring knowledge across different contexts found by Tirosh (1990). I would also argue that the probability scale has little value in the sense of developing meaning, unless it has numerical values. Children must link the probabilistic words to the mathematical concept i.e. In relation to a particular event, 'certain' means 100% or 1 on a probability scale and 'even chance' means 50% or  $\frac{1}{2}$ . Labelling 'even chance' as 'fifty-fifty' is also confusing because the child might view the whole term as relating to two different events e.g. If there is 50% chance of 'tails' occurring there is 50% chance of 'heads' occurring too i.e. the child views landing on heads, or landing on tails as an event rather than an outcome. Children need to know that 50:50 refers to the probability of one event having a possibility of two equal outcomes e.g. there is a 50% chance of tails and a 50% chance it is not tails when tossing a coin. Of course, if it is not tails then it must be 'heads' as the inverse of 'tails', which could be the root of confusion.

During Lesson 1, the teacher had recognised that children had different perceptions to his own because they gave subjective and intuitive responses to his questions. Lesson 3 was an opportunity to address the difficulties in

sharing a joint understanding of contextual references introduced to the lesson. Some difficulties arose because children had different views of the time (Extract 4.14) or place of an event.

#### **Extract 4.14 Time**

44 Teacher: Somebody who has a specific day for playing football at the park? ... A specific day...Jake.

45 Jake: Saturday

46 Teacher: Saturday, right then, let's just think about this for one minute. **If** I was saying to you Jake "where on scale would you put 'play football in the park' if you were talking about Saturday?" Where would you put it?

47 Jake: Erm... between 'very likely' and 'certain'.

48 Teacher: Right, so if it was Saturday and it was Jake's day in the park he would put it there. So, that's for that day. If I say to you... 'in general'. *On a general day "play football in the park".*

49 Jake: Erm... 'likely'.

Teacher: Right, so if we...we are using a specific time... if I give you a specific time, or say "in general". What do you notice? What do you notice? Are they the same?

51 Children: No

The teacher encouraged the children to **generalise** the contextual reference and to compare this to a specific time. Most children seemed to make sense of this situation as demonstrated by the class response elicited by prompting from the teacher. However, when later returning to the same idea, some children still had different understandings of how the time affected the prediction. Contextual references involving reference to a particular time seemed quite difficult for developing a shared understanding of meaning.

To summarise this strand of the analysis I shall identify the key points arising from each lesson observation. After the next section of analysis, I shall summarise the key points from observation of the structured interventions. In Chapter 5, I shall begin with clear summary of the findings

developed from considering the issues arising in both strands of my analysis.

#### Lesson 1:

- The teacher's response to the children's suggestions alters the contextual reference in order to alter the meaning to 'his' meaning and to make it shareable with the class.
- The teacher used pictorial images alongside verbal contextual references that had the potential to restrict the transferability of the meaning of a word across different contexts.
- The teacher had difficulty with controlling the 'open' dialogue with the children because of their subjective responses.

#### Lesson 2:

- Although the teacher had a specific focus on one phrase there was evidence of potential ambiguity in the use of different phrases with the same meaning, and in the introduction of different contextual references
- The teachers closed questioning technique might lead to the development of a narrow definition of even chance that is not transferable across contexts
- Numerical comparisons and experimental situations were much easier for the teacher to discuss with the class than 'real life' contexts, and the potential for sharing the same understanding of meaning seemed greater
- The teacher used different gestures, possibly sub-consciously, to exemplify the meaning of two different labels for 'even chance', thus creating the potential for different understandings of meaning developing in the class.

#### Lesson 3:

- The image of the probability scale likened to a time line by the children might negate the understanding of the scale as a continuum with limits, from impossible to certain.

- The placement of ‘real-life’ events on a probability scale was difficult because of subjective application by the children, and because too many variables exist making causal justification problematical.
- A child used the rocking gesture, introduced by the teacher in lesson 2, and there was evidence that some children had difficulty understanding the meaning of ‘even chance’, which they perceived as an either/or situation rather than a particular category.
- The lesson was useful in the sense that it provided an opportunity to address difficulties arising in previous lessons, but some children still exhibited difficulties with reasoning about ‘real-life’ events.

### **Analysis of the structured interventions**

I use a selection of different responses from different children to illustrate the variety of contextual references and their influence on the development of a shared understanding of meaning. My aim is to examine the ways in which different pairs communicated the sense they made of ‘even chance’ as a mathematical phrase. First, I look at ‘non-verbal communication’ and consider the effects of gesture and recording on paper, alongside the discourse they accompany. In particular, I am interested in whether these non-verbal communication strategies support the development of a joint understanding of meaning. Secondly, I analyse the influence of different contextual references on children’s reasoning processes. I describe these broadly as ‘everyday’ if they are derived from ‘out of school’ experiences, or ‘mathematical’ if they are derived from the conceptual content of the lesson. I use the categories described by Mercer (1996) for analysing peer talk. These are three types, **disputational**, **cumulative**, and **exploratory** as described in Chapter 2. These categories are most suitable for analysing the types of talk in relation to the use of contextual references. How a child responds to another child’s contextual reference might fall into one of these categories.

I noted the importance of gesture as part of communication occasionally in the lessons, but during structured interventions, some children relied heavily on gesture to help them communicate. McNeill (1985) said that gestures

help people to exemplify meanings of words through a synthesis of thoughts controlling the actions internally that produce external images of actions. My view is that the gestures emanate from a mainly sub-conscious image in the speakers mind. As words and gestures often appear together as one language action this process might be a sub-conscious response to the need to communicate effectively. The gesture provides the recipient of the message with a visual image alongside the words used. This visual image might then become a mental image in the recipient as a mediator for developing a joint understanding of meaning. Two children, Lee and Naomi used gesture to develop shared meaning in their dialogue (Extract 4.15). In the following extracts Res: signifies my intervention into the dialogue.

**Extract 4.15 Balance, symmetry and making even**

- 20 Res: Okay. What were you going to say Lee?
- 21 Lee: It's just like, say, erm... a number and a number and it's even, if you know what I mean
- 22 Res: A number...
- 23 Lee: And a number.... say ... it's even {Lee gestures with his hands as if imagining the numbers on a beam balance}
- 24 Res: Sophie you're saying it's got to be the same number...
- 25 Lee: Yes it's even. Yeah, the same as in any thing
- 26 Res: You're going like this {Res models the balance}...what do you mean?
- 27 Lee: I don't know
- 28 Naomi: It weighs the same. {Naomi models the balance and we all laugh}
- 29 Lee: Yes it's like that
- 30 Res: You're saying it's got to be like "balanced"
- 31 Lee: Yeah
- 32 Res: It looks [to me...
- 33 Lee: [even
- 34 Res: ...are you thinking about weighing scales [is that...
- 35 Naomi: [yeah
- 36 Res: ...what you're thinking? ... Yeah?

- 37 Lee: Yeah, that's what I thought of (inaudible)
- 38 Res: So you're thinking of the same number or the same amount
- 39 Lee: Yeah.
- 40 Res: On each side of something
- 41 Naomi: You put two weighs, 'weighers' in and then it's like  
 {Naomi's hands are doing the same movements as if in a mirror}
- 42 Lee: [and they are the same...{Lee gestures balancing scales}]
- 43 Naomi: [and put two...{Naomi gestures mirror symmetry}]
- 44 Lee: ... it's the same chance, even. They're even. {Lee gestures a  
 symmetrical flattening or 'making even'.}

Lee used gesture to support his idea that 'even chance' is 'the same' and he thought in numerical terms, but visualised the relationship as a balance (23). He used the idea of balance as a metaphor for even chance, demonstrating something unique from the other contextual references. I **refined** his response to clarify that he was talking about the same number. Naomi supported his idea, extended and developed the meaning in **cumulative** dialogue and associated gestures. Again, I **refined** the responses and introduced the word 'balanced'. Her gestures later included mirrored hand movements when putting 'weighers' in the scales (41,43), and symmetry then appears in Lee's gestures when exemplifying the meaning of 'even' (44). Although the use of gesture was very important to the development of a shared understanding of meaning, Lee did not transfer this form of imagery into drawing, instead preferring to use the symbols  $\frac{1}{2}$   $\frac{1}{2}$  for 'half and half' (Appendix 8, Lee).

Stuart and Tessa provided a lot of support for each other through gesture and facial expression. They sometimes negotiated turns through gesture. Their talk was an interesting mixture of **disputational** and **cumulative**. Stuart sometimes showed **exploratory** talk with himself (also noted in the pilot). Both seemed willing to take risks and present ideas. This might be because they had worked with me before and knew that I would value their ideas. Stuart began to explain 'even chance' by referring to a football tackle, a

contextual reference originally introduced by another child in lesson 2 (Extract 4.16).

#### **Extract 4.16 Football tackle**

1     Stuart: Even chance...mmm {looks doubtful}... I know {hand up to stop Tessa}...even chance... right... even chance means right...say like, there's two people at football, and the ball's there, and they go into a tackle {demonstrates with fingers}, and its 50:50 {flat horizontal movement with one hand} it's an even chance like {two hands 'weighing' or balancing}, they can get the ball.

The similarity between Stuart's gestures and those in the previous extract (4.15) are clear, although used alongside different words. It is interesting that two different children introduced the 'balancing' gesture, like a bucket balance, as a metaphor for 'even chance', although the teacher had not introduced it in the lesson. Gesture seemed to be an essential part of communicating meaning here. The teacher had used the flat horizontal movement once, during Lesson 2 when he spoke about 'even chance'. He also used a small rocking movement with one hand when to illustrate 'fifty-fifty'. Such gestures that accompany our speech are not always deliberate. We might not consciously decide to use a particular gesture to exemplify word meaning. A gesture might be an indicator of a person's own perception of the word. In my own experience, a person that has difficulty explaining something seems to gesticulate more. People particularly seem to gesticulate when describing movement, or position (McNeill, 1985). Children who respond to visual 'images' associated with words easily imitate gestures, quite often sub-consciously. It is not surprising therefore, to note children using gesture in a similar way to the teacher. In both pairs, the recipient of the message copied the gesture, as if it helped to reinforce the intended meaning. Unlike Lee, Stuart supported his explanation with a drawing of two footballers at equal distances from a football (Appendix 8, Stuart).

#### **Extract 4.17 Interpreting drawings**

6     Res: Are you saying they are an equal distance from the ball?



- 7 Tessa: That ones nearer to the ball.
- 8 Stuart: It isn't 'cos I've got three lines...I'm not going to measure it and everything am I?
- 9 Res: Right, so three lines means it's the same distance?
- 10 Stuart: Yeah
- 11 Res: Yeah so what does it mean? That's an example.
- 12 Stuart: 'Even chance' means like you've both like, got a 50:50 chance you know like (laugh)...mmm...like {Stuart gestures with hands flat like scales going up and down, balancing. Tessa copies gesture and smiles to Res}

Tessa **challenged** the pictorial representation after I questioned to establish that Stuart thought 'even chance' in a tackle meant players at an even distance from the ball. The picture seemed to be an interpretation of the phrase 'even challenge' as introduced into lesson 2, by the teacher (Extract 4.8). An even distance from the ball is only one variable that influences a player's chance of winning it. The centrally placed ball, equidistant from the players is a similar image to the centrally placed category 'even chance' along a probability line. Although the children did not use a probability line in their explanations, their visual perceptions of the line from Lesson 3 might have influenced their gestures and other pictorial representations. The centrally placed prediction of 'even chance appears visually like the fulcrum in a balance, especially when an arrow signifies the position. If this is what children perceive, then a shared understanding of the meaning of 'even chance' in relation to the scale as a continuum is unlikely. The children appear have followed a learning process in which they have experienced both action and language during the lesson that helped them to make sense of the situation (McNeill, 1985). What is unclear is whether they have any mental images developing as part of the process. The act of gesturing, writing or drawing might be an indication of the 'image making' and 'image having' process that Kieren (1993) proposed as discussed in Chapter 2.

The idea of equal parts, or measures continued to influence Stuart's thoughts about 'even chance' when discussing the true or false statement (Extract 4.18). The children previously discussed the chances of different football

teams and this led to specific discussion about the goalie and the size of the goal net. Tessa thought two footballers would have a 'fifty-fifty' chance of getting a goal because they both trained for the same team originally (37). As I have discussed in relation to the lesson observations, providing reasons for predictions of outcomes real life events is difficult, unless some numerical data exist. Stuart described an image of the net and the goalie that suggested the goalie is blocking some of the chance so it reduces to 45, but he is not certain that his own thoughts are valid (44).

#### **Extract 4.18 Maybe a 45?**

- 36     Stuart: Oh! David Seaman
- 37     Tessa: Yeah, right he was in net... and they would have a 50:50 chance because they got trained both for Man U.
- 38     Stuart: No they wouldn't because you know you said 50:50 chance in an open net Well now it's a little line in the centre like a little man {drawing}
- 39     Tessa: I don't get him {looking at Res.}
- 40     Stuart: It's like putting a stick in the centre.
- 41     Res: What's like putting a stick in the centre?
- 42     Stuart: It's like there's an open goal right a you've got a skinny man – you've got a skinny man in the centre and...
- 43     Tessa: I get him, ha ha.
- 44     Stuart: ... and that blocks a little bit of you're chance so it'll be like ...I dunno erm...Maybe a 45, no not a 45 ... its got to be an even chance ... but it can't be an even chance 'cos...

It seemed that Stuart was relating chances of getting a goal to a measurable quantity. Such a measure might be one variable, but there are many others in such a real life context, so his reasoning was **exploratory** as he verbalised his ideas. Tessa initially could not understand, so I asked for more detail. She then seemed to make sense of his idea, but neither of them were able to make a judgement about the chances of getting a goal.

Diane communicated contradictory ideas about the same event in a similar way to a child described by Pa (1991). First Diane talked about numbers higher or lower than three and then described them as odd numbers, for the

same event (Extract 4.19, 2 and 6). She was assuming that 3 was the central point in the series. In order to clarify what she meant by higher or lower than three I asked what six numbers she was talking about. I then elaborated on her response in an attempt to clarify meaning (7), but might have caused her to change her original meaning as she agreed with my elaboration.

#### **Extract 4.19 Six numbers**

- 2 Diane: 'Even chance'...erm ... right, if you have erm... like six numbers, then three would be half of it and it would be an **'even chance'** that you might get lower than 3 or higher than 3.
- 3 Res: Right, can you draw a picture? Would it help you to draw that?
- 4 Diane: O. K. yeah. On there? {Draws}
- 5 Res: Yes ... So, what are the numbers on each side?
- 6 Diane: Erm... odd (inaudible)
- 7 Res: Right, so you're saying there that it is an 'even chance' that you'll get and odd number or an even number.
- 8 Diane: Yeah.

Her picture showed three circles on either side of a line that she labelled 'even chance' (Appendix 8, 3 Diane). This might be evidence that Diane had difficulty explaining clearly in her own words and when prompted provided what seemed initially to be a more accurate answer. The drawing however, is a symbolic representation of a set of six numbers. Her answer 'odd' might be a reference to the fact that there are three circles on each side of the line. The symbolism is representative of two equal sets on either side of a line. Another child drew a similar linear model (Appendix 8, 7 Jan). Along with the previous example of two footballers, these provide pictorial evidence that the children thought in terms of equal sets, or amounts. Their pictures suggest 'balance' rather than the placement of 'even chance' along a continuum. Diane's picture did not seem to exemplify her original explanation and therefore was not a useful part of communicating meaning. The experimental activities in lesson 2 influenced many of the children through use of number to exemplify 'even chance'. Jake cited an example straight from the lesson (Extract 4.20, 20)

**Extract 4.20 Equal numbers or amounts**

- 20 Jake: If you've got like three red counters and three blue counters it's an 'even chance' because they've both got an equal number.
- 21 Res: An equal number of things. So, what are you saying about 'even chance'? For something to have an 'even chance' what has it got to have?
- 22 Jake: The same.
- 23 Res: The same what?
- 24 Jake: Amount
- 25 Res: The same amount or the same number... {Jake nods}

My question is an attempt to encourage Jake to make a general statement, and I **refined** Jake's response. However, describing 'even chance' as equal sets or amounts sometimes seemed to lead to misunderstandings. Steinbring (1991) noticed that teachers often narrow down the options in probability and therefore limit the children's ideas.

Stuart from claimed that a draw was 'fifty-fifty' presumably because of the equal number of goals scored (Extract 4.21). Other children exhibited similar reasoning.

**Extract 4.21 A draw equals 'fifty-fifty'?**

- 46 Tessa: False
- 47 Res: You're saying false. Why are you saying false Tessa?
- 48 Tessa: Because you might draw or something like.
- 49 Stuart: Yeah but that's 50:50 chance

Stuart focused on the equality of two sets to indicate 'even chance' regardless of the fact that in the context of a football draw this was a wrong assumption to make. Ryan and Holly, talk about 'even chance' as splitting in two or sharing (Extract 4.22, 2 and 4). Again, they are focusing on the equality of sets without considering whether such an assumption fits all contexts.

### Extract 4.22 Sharing

- 2 Ryan: Even chance is like say...say...if you like...you might not die and you might...so it's like...
- Even chance is like erm, say if there's 50 and 50 makes up a 100 doesn't it? Split that into a half. If you split 100 into half you make that even don't you? Sophie it depends really, if you are going to die or not...I think that's what it means anyway.
- 3 Res: So you're saying its like 50:50. If you've got a 100...
- 4 Holly: *Then you're sharing...*
- 5 Res: ...and you split it into 50 and 50 then they are even. Yeah? Can you draw a picture to show what you mean?

In the next section, children focused on another important mathematical concept, that of equally likely outcomes, but without recognising that an event demonstrating 'even chance' has only two equally likely outcomes. Such recognition is currently an objective at Year 6 (DfEE, 1999a) so we might not expect children in Year 5 to develop a secure understanding of the meaning of 'even chance' in terms of outcomes. Nita and Jake, discussed the true/false statement for 'probability'. Both spoke to me rather than each other. Nita was particularly reticent until this point (Extract 4.23)

### Extract 4.23 Numbers on a die

- 2 Jake: It is more difficult to throw a six or a one than any other number, on a die.
- 3 Res: Do you know what the die is?
- 4 Jake: A dice.
- 5 Nita: That's false, because ... it is an 'even chance' and, ... you've got and then because it's one number...{Nita gestures on the table with a circular motion while talking} I mean... all the numbers are once, on the dice, so... it's not like you are not going to get six or one on the.....
- 6 Res: Right so what you are saying is there is only one of each number on the dice it's just as easy to get six or one as any other number. Would you agree with that yeah?{Jake nods}

Another child's reference to dice during lesson 2 (Extract 4.10, 18) might have influenced Nita (5). Nita's answer contradicted her correct response in lesson 2 (4.10, 20). However, in lesson 2 the teacher had requested an event with two outcomes only. A clearly focused question resulted in a clearly focused and correct response during the lesson. Later in the discussion, Nita describes a situation that would not be 'even chance' (Extract 4.24).

#### **Extract 4.24 Changing outcomes**

- 12 Nita: 'Even chance' is where, say if you had a dice and it's got six numbers on and if you roll it it's an 'even chance' that you'll get erm 1 2 3 4 5 or six. But, if you had a dice with the like two two's on it or something it wouldn't be an 'even chance' because you'd be more likely to get a two.

There is clear evidence that Nita made sense of 'even chance' in relation to equally likely outcomes, but she was unclear about the fact that it is between two outcomes only. She had a clear sense of the numbers available on the die affecting the outcomes.

The effects of 'everyday' contexts became evident in several structured interventions. Children seemed to perceive relationships between ideas as transferable when they were not, as in the example where the child suggested a draw was 'even chance'. Alternatively, because of the different contexts the children did not transfer ideas. Holly, thought that having a 'chance' was having an 'even chance' in the context of a race. This is similar to the observation by Steinbring (1991) where he noticed that teachers might give the impression that 'chance' is associated with any prediction that is not certain. A child might easily think that any chance in a race is an 'even chance' because everyone starts from the same line. This is a reasonable response in an everyday context because the reason a race begins from a starting line, at the same time, in the same conditions is to give everyone the same chance. However, it is also true that some people can run faster than others can, and that the qualities possessed by the competitors are variables for consideration in relation to chance. I probed

Holly's ideas, by asking her to think of an alternative scenario (Extract 4.25). Holly recognised that the qualities of the competitors was a factor to consider. I **refined** her response in order to clarify meaning.

#### **Extract 4.25 Developing reasoning**

- 27 Res: Let's go back to the race because you said in a race you have an even chance of winning. Is there a time when you might not have an even chance to win?
- 28 Holly: If you were racing against fast people and you were dead slow, that would not be very even, 'cos you would have to have all the slow people, then all the fast people in a race.
- 29 Res: Right, so if you went in a race where there was somebody who was known to be very, very fast and you know that you wouldn't race that fast, you're saying that wouldn't be an 'even chance'? {Holly nods}

The above dialogue was part of a discussion about the true/false statement for 'even chance', which also included comment that each team had the same chance. The discussions about the chances of various football teams chances of winning highlighted the difficulties of working with everyday contexts when trying to make sense of probabilistic language. In the next extract, Nita talked to Jake rather than through me (Extract 4.26, 34). The continued opportunity for extended discussion time might have increased her confidence to address her partner (Johnson, Hutton and Yard, 1992).

#### **Extract 4.26 True or false?**

- 30 Nita: It is an 'even chance' for each team to win the match.
- 31 Jake: Well it depends if... like...'cos...one team might have better players...
- 32 Nita: [True
- 33 Jake: [so it's true or false... it's gonna be in between.
- 34 Nita: No I think it's true because every team's got...like...a chance to win there might be better players on another team but you've all got a chance to win like

- 35 Res: What are you saying Jake?
- 36 Jake: You might have better players... or you might have no sub if someone gets injured so you've got less players.

Jake thought it was in-between while Nita **challenged** his idea, providing her own reason for thinking it was true. My intervention was to draw out the different reasons for Jake's choice. After further probing, he provided even more reasons. Jake clearly had ideas to share, but my intervention was necessary to elicit them. Nita showed again that she had difficulty making sense of 'even chance', beyond the unambiguous numerical context. She even described the lottery as "sort of even chance" (Extract 4.27, 55), perhaps influenced by Jake's gesture (54) although he suggests 'likely'. Later, she supported Jake's reasoning for 'likely' rather than 'most likely' by saying there are 49 numbers

#### **Extract 4.27 The lottery**

- 54 Jake: I'd say that (inaudible) {a rocking motion with his hand}
- 55 Nita: It's sort of like 'even chance' isn't it?... Because somebody might win but it's not always certain that they will.
- 56 Res: Right... okay, so you're saying that...
- 57 Jake: I'd say it was 'likely'.
- 58 Res: You think it's 'likely' someone will win rather than 'most likely'... Why?
- 59 Jake: Because, like, you've got...not got a lot of chance of getting because...
- 60 Nita: There is 49 numbers
- 61 Jake: And you've only got six

Jake, however, focused on the likelihood of a particular person winning rather than 'someone'. Most weeks, someone does win the lottery, and so it is most likely, but he is thinking of one person and this idea lead to his decision of 'likely'. This in itself is illogical, because most people are unlikely to win. The dialogue shows the same difficulty that the teacher and many children encountered in lessons 1 and 3 when separating out the



broader situational context, from the more individual context. There seems to be some difficulty in making probabilistic judgements across different contexts, thus making transfer of learning problematical (Tirosh, 1990). This difficulty might arise because of the 'every day' nature of probabilistic language, and I previously noted a similar difficulty with transferring meaning across different contexts in lesson 3.

Sally and Tim, were most confident and demonstrated willingness to debate ideas, but there was lot of **disputational** talk, with one trying to outwit the other. They had lengthy exchanges, but there was evidence that Sally was not a good listener. She considered the activity a challenge and the camera affected her response. When talking about 'most likely' Sally insisted that she asked a question (Extract 4.28 ).

#### **Extract 4.28 Stopped by a train**

1. Sally: Yeah, I've got a good one. How do you know that it's not 50:50 and how do you know it's 'most likely'? {taps table with pencil}
2. Tim: Because it comes every two minutes or so.....
3. Sally: But what if it doesn't come every two minutes?
4. Tim: You said it came every two minutes
5. Sally: Yeah, I was just doing that as a...example
6. Tim: Say it comes every two minutes, right? Every two minutes. It comes along.... and you are 'more likely' to...two minutes isn't as long as two hours is it?
7. Sally: No
8. Tim: So you are 'more likely' to be stopped by the train coming because it doesn't come every hour or so, it comes...it's a shorter time. {Tim points to the line on the table. He looks at Sally}.It's a shorter time.... between every one. So if it ... so you'll...
9. Sally: Yeah but it's got a **50:50** chance, because it's got one minute then another minute, {hands on table, one and one, then as if drawing a line between the two} so if you are one minute late you've got... If you are one minute late you usually get there before

the train comes you're gonna catch it, but you've got a **50:50** chance, because you don't know if you're going to get stuck in traffic on the way there.

10. Tim: Fine.

The contextual reference of "being stopped by a train" proved to be a difficult one for discussing probabilities. Tim used it as an example to describe ideas about 'most likely' but when Sally tried to clarify meaning with him, he began to talk about 'more likely'. His reasoning for 'more likely' was good, but he did not attempt to justify why it was not 'fifty-fifty'. It is interesting that Sally thought a train coming every two minutes meant 'fifty-fifty' chance. Perhaps she thought she could miss it in one minute and it would stop her in the next. Later in the discussion, she claimed a train once a week was 'fifty-fifty'. I have included this particular extract to show that children's willingness to enter into peer discussion, does not necessarily lead to a shared understanding of meaning, nor develop sound mathematical reasoning. For this particular pair, adult intervention would help to clarify lines of thought about the nature of 'even chance'.

From my analysis of the structured interventions, the following key points are evident. I will collate these with other key points from the analysis of the lesson observations in order to provide a summary of findings at the beginning of Chapter 5. These will provide the basis of my conclusions in relation to my original research questions:

- Some pairs of children were more able, or willing, to enter an educated discourse than others.
- A willingness to enter into peer discussion did not necessarily support the development of a shared understanding of meaning.
- Non-verbal communication, gesture, drawing or both, was an important communication strategy for some pairs.
- The images suggested by gesture and drawing might not be very useful in developing a shared understanding of meaning of mathematical

language. The gestures used, and drawings produced for 'even chance' generally suggested balance, evenness and equal shares.

- Some of the children's reasoning shows that they might have a perceptual view of 'even chance' as equal measures, or numbers.
- References to 'real-life' events were not useful for developing a shared understanding of meaning, and did not develop children's probabilistic reasoning skills effectively
- The experimental work in lesson 2 possibly led to some children applying numerical reasoning inappropriately
- The children did not refer to a probability scale when discussing 'even chance'.
- Children who had little understanding of the meaning of 'even chance' as two equally likely outcomes could not communicate this effectively through the examples they chose.

#### **Summary of Chapter 4**

In this chapter, I analysed the different communication strategies in the lessons and structured interventions. Each strand of analysis is summarised as key points (pages 113 –114; pages 127 – 128) that help to answer my original research questions. Chapter 5 uses these key points to generate a summary of the overall findings and subsequent discussion of the educational issues arising from them.

## **Chapter 5: Conclusion**

This chapter has five sections. First, I summarise the findings that have emerged from the analysis of both strands of my research. In my second section, I consider the educational implications of my findings, in particular the teaching of probabilistic language in the primary school. My third section identifies the contribution my research has made to the theory of teaching and learning. In the last two sections, I evaluate my research methods and consider future directions emerging from the study.

### **A summary of the findings from both strands**

My research explored two main questions focusing on language use in mathematics teaching and learning:

- 1) How does the teacher interact with the children to develop a shared understanding of the meaning of mathematical words?
- 2) How do children express their understanding of the meaning of specific mathematical words and phrases?

Although two different research strands, lesson observations, and structured interventions set out to explore the issues of teaching and learning separately, they are interrelated. Each lesson observation provided slightly different data because each lesson was different in nature. The data from the structured interventions shows the lessons' influence on the children's communications. At the end of each strand of my analysis in Chapter 4, I have identified a list of key points. I have summarised these findings below, drawing together any common issues where relevant. I justify my claims through reference to relevant texts in Chapter 4, and in the second and third sections of this chapter when I consider the educational implications of my findings and their contribution to the theoretical understanding of the processes of teaching and learning.

The teacher's aim was for the children to develop an understanding of the meaning of probabilistic words. He interacted with the children using

questions, and reformulating answers in an attempt to develop common knowledge. In particular, he tried to establish a common meaning for the various contextual references made during the dialogue, or in the school text. He used specific pictorial representations and practical models as part of his deliberate teaching strategy to communicate meaning. However, the lessons contained several areas of potential ambiguity when considering the development of a joint understanding of meaning. In his interactions with the children, the teacher tended to alter their responses to suit his own 'meaning'. He aimed for the children to develop the meaning he intended. When he invited ideas from the children, he reformulated their answers in different ways that did not seem to improve the potential for reaching a shared understanding because children then challenged his ideas. By inviting the children's suggestions, for events to fit into particular probabilistic categories, the teacher felt he lost control of the direction of the dialogue and therefore could not enable shared understanding of meaning to develop. Children viewed probabilistic events from a subjective or intuitive point of view and therefore used the language in different ways than expected. When he used 'closed' questions requiring specific answers the communication was less ambiguous. He 'narrowed' the referential context by using numerical experimentation, rather than 'everyday' contextual references in order to establish meaning for 'even chance'. There was the likelihood of limiting the transferability of meaning across different contexts, and this became evident in the structured interventions. Gesture was also part of the communication process that affected a shared understanding of meaning of the phrase 'even chance'. During the lesson, little opportunity existed for children to reconstruct new meanings through discussion. It was evident from the lessons and the structured interventions that many of the children were unused to entering into educated discourse with their peers.

*The pairs of children placed different emphasis on different ways of communicating, most likely due to different levels of confidence and skill in spoken communication. They used 'cumulative', 'disputational' and some 'exploratory' talk to different degrees. Cumulative talk, where children*

built on each others ideas seemed particularly effective in generating an agreement about the meaning of a particular word or phrase. Two pairs of children relied heavily on gesture, while another pair used drawings as part of the communication process. The meanings they gave to particular words such as 'even chance' varied according to the contextual references used, and such references often created difficulties because the recipient had a different meaning than that intended by the speaker. The use of pictorial representations, and occasional gestures also appeared to impact on the development of a shared understanding of meaning. Some of the pictorial images used by the teacher during the lessons were confusing, in relation to the use of language and the mathematical concept of probability. The children's explanations and their limited reference to the probability scale reflected this. Gestures used by the teacher in association with particular words or phrases in the lessons, also seemed to affect the meaning that some children gave to them. Some of the children relied heavily on similar gestures, and various drawings, as part of their explanations during the structured interventions, but there was little opportunity during lesson dialogue for children to use gesture and drawings when explaining their ideas to the class. The pictorial representations from both teacher and children often supported spoken references to 'everyday, pseudo real-life' events for which the probabilistic outcomes were difficult to predict. The teacher and children used probabilistic words as labels to predict outcomes, such as 'even chance', together with pictorial representations and gestures. This labelling process possibly created a limited understanding of the mathematical meaning, thus affecting transferability. Discussion and prediction of numerical outcomes was much easier and seemed less ambiguous than referring to 'everyday' events, but during the structured interventions children demonstrated that their understanding of the meaning of such outcomes was limited as they tried to apply it inappropriately.

### **The educational implications of my findings**

I will first consider the implications of my findings for 'whole class interactive teaching' as recommended by the NNS (DfEE, 1999a). Most important are the implications for teaching probabilistic language.

Following this, I will consider the implications for teacher training, where trainees follow a curriculum that expects them to learn how to use a precise mathematical vocabulary.

The National Numeracy Strategy describes a three-part mathematics lesson with 'whole-class interactive' teaching as discussed in Chapter 2. Teachers are encouraged to use a variety of approaches including modelling, explaining, demonstrating, discussing and questioning to teach mathematics to the whole class. Children should be encouraged to take an active part by explaining and sharing their mathematical ideas through correct mathematical language. Although I observed lessons before the implementation of the NNS in the school, they generally followed the three-part structure, and the teacher used a range of communicative strategies. The strategies used however, led to children developing unclear meanings of probabilistic language. Fischbein *et al.* (1991) also identified that children developed unclear meanings of probabilistic language, but they did not identify any reasons for this. Steinbring (1991) identified that the structured nature of probabilistic lessons affected the development of meanings, especially in developing the idea that predictions such as 'most likely' are interpretations of events based on a specific set of current knowledge. Predictions alter according to influences such as place, person and time. In my research, certain communicative events provide the reasons for this apparent lack of clarity, identified during my analysis. It is necessary therefore to consider the effects of these communicative events. If a word acquires its properties or meanings by association with events as suggested by Wallwork, (1985) then it seems logical to assume that if the meaning is unclear then the events must have been unclear. In the observed lessons, the following communicative events had the potential to create a lack of clarity for children:

1. Children's answers to questions that referred to 'everyday' events
2. The teacher's reformulations of those contextual references
3. Pictorial images intended to support the development of the meaning of probabilistic language

#### 4. Gestures accompanying the phrase 'even chance'

Children's answers to questions often included contextual references that might not fit with other children's knowledge and understanding of the world. Vygotsky (1987) said that words only make sense in context, and that they change sense in different contexts although the meaning stays the same. If this is the case then perhaps the children's answers made sense to them in the context they referred to, but for other children this made different sense. This could be for two reasons. Either their previous understanding of meaning of the mathematical word fitted into another context for them, or they had a different understanding of the context referred to. In recognition of this, the teacher's reformulations were an attempt to change the contextual reference to one that he thought everyone would make sense of and to move the children towards his intended meaning. In some cases, reformulations were a control mechanism, when the teacher had difficulty maintaining the flow of the lesson due to the subjective ways that children used probabilistic words. Unfortunately, the result of his reformulations generally meant a narrowing of options and reduced opportunity to understand the tentative nature of probabilistic language, also identified by Steinbring (1991). The implications for whole-class interactive teaching are that teachers cannot assume that all children develop the same understanding of meaning through such an approach. There is often a mismatch between the 'intended meaning' of the speaker and the meaning received by the recipient, as described by Bakhtin (1981). To support their teaching primary teachers might also use texts that promote the use of 'everyday' contexts with the purpose of teaching the children how to use probabilistic language. It appears from my research that such use can be confusing, especially for predictions of an event having two equally likely outcomes, or 'even chance'. Teaching probabilistic language in primary school without regard to the later development of probabilistic concepts at secondary school will most likely create misconceptions about the nature of chance. Using 'everyday' contexts might add to these misconceptions unless they are chosen carefully, and children have a range of opportunities to discuss them.



My findings also indicate that whole class discussions rarely provide opportunity for children to construct a shared understanding of meaning through dialogue. There are too many participants and too many opportunities for misunderstandings to arise. I agree with Barnes (1992) that lessons must include opportunity for paired and group discussion allowing exploratory talk between peers to reinforce the development of meaning. This is particularly true of probabilistic words because 'chance' is often transient in nature, as described by Steinbring (1991), and peer discussion ought to help children clarify the range of issues and begin to understand the tentative and transient nature of the predictions they make. The NNS training materials (DfEE 1999d) recommend paired and group work as part of the lesson, so children ought to be able to develop a shared understanding of meaning, during their 'daily mathematics lessons' following NNS recommendations. In reality though, many teachers will face difficulties in coping with the tension between providing opportunities for children to develop ideas through discussion, and the need to ensure they teach a particular objective. Effective teachers are those who interact with children at all times and in all types of lesson situations (Hay/McBer, 2000). In mathematics lessons, such levels of interaction are very challenging for a teacher who might not have the specialist knowledge to support it.

Pictorial representations of probabilistic concepts ought to match the tentative nature of the language involved. For example, the chart in lesson 1 labelled events as distinct outcome categories, contradicting the tentative nature of predicting the probability of real life events and the nature of the probability scale as a continuum with specific parameters. I can only speculate on the effects of such images in my research but it seemed evident, as I have identified in Chapter 4, that the children might have developed a mental image based on the first lesson that affected their perception of the probability scale in the third lesson. This was not the only influence because they also associated the scale with a time line from a previous history lesson. I agree with Pendlington (1999) that supporting the development of imagery is important in lessons, and NNS recommendations include the use of a range of resources for that reason. However, my research indicates that

teachers require greater awareness of the effects of any pictorial representations they provide. Such representations are part of the ‘image making’ process that Kieren (1993) describes, but we cannot assume that the mental image produced by the recipient holds the same meaning. Teachers therefore need to consider carefully, in mathematics lessons, whether the pictorial representation supports the development of a shared understanding of meaning of words in relation to correct use of mathematical concepts and ideas. The effects of these representations over a series of lessons also requires a clearly thought out plan for successful linguistic and conceptual development. For Year 5 children to understand the nature of probabilistic events as described by the recommended language in the NNS Framework (DfEE, 1999a) there appear to be three possible recommendations I could make, based on my research:

1. Avoid the use of ‘everyday’ contexts when teaching about ‘even chance’.
2. Use the probability scale as the main pictorial and linguistic representation for probabilistic concepts.
3. Ensure opportunity for peer discussion about events.

The first of these recommendations seems to contradict exemplary material provided in the NNS Framework Years 5 and 6 (DfEE, 1999a Section 6 page 113), which contains several examples using ‘everyday’ events. However, the Framework presents examples that are representative of particular categories that provide opportunity for some discussion without too much ambiguity. Although ‘even chance’ is included on the probability scale in Year 5, there is no example of a real-life event to fit this category. It is there to signify the central point on the scale in relation to identification of ‘more likely’ and ‘less likely’ events. In Year 6, the Framework advises teachers to ‘discuss events which might have two equally likely outcomes’ and provides appropriate examples. Thus, the first recommendation might be better stated:

1. Choose everyday events for which there is a likelihood of two equally likely outcomes to happen, when discussing 'even chance'.

In a similar way I can identify that the Framework presents a pictorial representation of a probability scale for Year 5, that shows the tentative nature of probabilistic events, linking the categorical labels such as 'certain, good chance, even chance, poor chance and no chance' with 'less likely and 'more likely' as a continuum (DfEE, 1999a, Section 6 page 113). My recommendation to use the probability scale as the main pictorial and linguistic representation for probabilistic concepts is too simplistic, because teachers must ensure they use it to develop understanding of the mathematical concepts alongside the language. Therefore, my recommendation is not just to use the scale, but also to ensure it aids both linguistic and conceptual development. I have developed his recommendation further:

2. Use the probability scale as the main pictorial and linguistic representation for probabilistic concepts, ensuring that Year 5 children learn that the set of labels placed along the probability scale are points on a continuum from the fixed category 'no chance' or 'impossible' at one end, to 'certain' at the other.

I can also refine my third recommendation for Year 5 because providing the children with opportunity for peer discussion without adult intervention might not support the development of a shared understanding of meaning. This became evident during the structured interventions when I had to intervene in order to keep the dialogue focused, or to clarify meaning. Peer discussion was most effective when it included cumulative exchanges as described in Chapter 2 with reference to Mercer (1996), because disputational talk and exploratory talk appeared to cause more ambiguity that the children could not resolve alone. Teachers or another adults therefore need opportunity to clarify ambiguity arising in peer exchanges. Within the three part mathematics lesson such opportunities arise either during the main part of the lesson, when paired work is part of the

introduction to the topic, or during the plenary where the teacher draws together common ideas from the class. My third recommendation is therefore:

3. *Ensure appropriately supported peer discussion about predicting the outcomes of events*

My recommendations show clearly that my research indicates that learning probabilistic language is closely associated with learning probabilistic concepts. I did not set out to explore this relationship, as I was focusing on the communication processes, but eventually, through analysis, the necessity to use the words accurately in relation to the mathematical concept became clear.

I was initially surprised at the possible impact of gesture on the communication of a shared understanding of meaning of mathematical language. My first viewing of lesson 2 did not highlight gesture as an important communicative event, but the structured interventions did. The teacher used small but clear gestures suggesting 'balance' and 'evenness' for the terms 'fifty-fifty' and 'even chance' that were barely noticeable. I suggested in Chapter 4 that these particular gestures appeared to be sub-conscious, i.e. the teacher did not seem to have made a deliberate choice in using them. These gestures appeared to affect some children's understanding of the phrase 'even chance', as four pairs of children used such gestures, with two pairs relying heavily on gesture. The children relying more heavily on gesture to describe 'even chance' seemed to be less able to describe it in words alone, so the gesture had a clear communicative function as described by McNeill, (1985). During one structured intervention a third type of gesture arose, that of symmetrical movements of the hands, suggesting 'equality' and perhaps evidence of the child's perception that 'even chance' means equal chance. Hence, there was a strong perception, communicated through gesture, and speech, that 'even chance' meant balance, evenness, or equality. The children's discussions, drawings and writing suggested similar perceptions. These perceptions might also negate making sense of the nature

of a probability scale as a continuum with fixed parameters. Based on these observations I will also recommend that:

4. Teachers regard gesture as an important communicative event, especially when children are trying to explain their understanding of mathematical words and concepts.

Obviously, teachers cannot control their own sub-conscious gestures, although they can control their deliberate gestures, just as the teacher deliberately used gesture in lesson 1 to elicit a response from a child. However, they can make themselves more aware that their gestures, unknown to them at the time, might have an impact on the development of meaning for the children. Teachers can also observe gestures that children make, and encourage children to use gesture when they have difficulty expressing their understanding of meaning in words. My experience in the structured interventions suggested that encouraging such communication, with support from the teacher, can lead to children's improved ability to use and express their ideas in words.

All of the above recommendations also apply to initial teacher training courses. Trainee teachers still have to meet the initial teacher training standards discussed in Chapter 1 with regard to precise use of mathematical language, although initial teacher training curricula are currently under review. For teaching probabilistic and other mathematical language, trainees require an understanding of 'precision'. In probability there is a danger of applying the notion of precision to phrases such as 'most likely' when this is not a precise category in reality. Probabilistic predictive categories that one *might* consider precise in nature are 'certain', 'impossible' and 'even chance'. I claim this because an event is either 'certain' or it is not; 'impossible' or not; 'even chance' or not. However, one still cannot make predictions about these events easily, as Jones *et al.* (1997) found when working with children. I identified in the analysis that Year 5 children applied subjective and intuitive interpretations to events. We must still use 'most likely' in a correct manner in a mathematical context and so we

demonstrate 'precision in use', rather than in the properties of the concept that the phrase describes. '**Precision in use**' means correct use in relation to the mathematical concept. When predicting an event to be 'most likely' a certain amount of subjectivity will affect one's perception. Children might view 'most likely' as 'almost certain', or just a degree more than 'likely'. As identified in Chapter 4, the teacher commented that children found comparative predictions much easier. Primary trainees require guidance in the use of the NNS framework material for teaching probability, and an understanding of the progression into Key Stage 3, so that they can best introduce the language and develop a shared understanding of meaning that is precise in relation to the dynamic nature of probabilistic concepts and effective grounding for children's future learning.

### **Contribution to theoretical understanding of the processes of teaching and learning**

My research contributes to current theoretical understanding of teaching mathematical language in several ways. It supports a number of the issues about language and teaching raised by the various elements of the National Oracy Project as reported in Norman (1992). In particular, I have identified difficulties in developing a shared understanding of meaning through whole class teaching, even though the teacher set out with clear objectives and followed a lesson structure similar to that described by Jarvis *et al.* (1997). Jarvis *et al.* suggested that the structure they identified only has the potential to make meaning shareable, but not guaranteed and my findings support this. In Chapter 1, I discussed research by Bell *et al.* (1983) that informed the Cockcroft Report in 1982. The report emphasised the importance of discussion in the mathematics classroom as a means of learning to communicate mathematically. Whole-class situations do not lend themselves to true discussion, and limit the opportunity for exploratory talk, and collaborative talk, that can occur more effectively during peer talk as reported by Barnes (1992) and Wood (1994). Thus my research supports views of other researchers in identifying the need for children to develop an 'educated discourse' described by Mercer (1995) through opportunities for discussion. My research has also highlighted the difficulties some children

had in expressing their understanding of meanings without using drawings or gestures to support and develop their explanations. It seems logical, in retrospect, that if teachers use a variety of communication strategies, including use of pictorial representations, then children need opportunity to do the same. Currently gaining popularity in primary mathematics classrooms is the use of hand-held whiteboards on which children could, if encouraged to do so, express their mathematical ideas in pictures. However, in my experience, such boards are often an informal assessment opportunity for the teacher, rather than an opportunity to generate 'educated discourse'. From my observations, I suggest that educators regard gesture and drawings as communicative strategies that enable some children to participate in both *educational* and *educated* discourse, when otherwise they would not.

From conducting a pilot study about fractional language, and then the final study about probabilistic language I have identified that some common issues exist, relevant to the teaching of mathematical language in general. However, there are also specific issues related to the conceptual development within probability, which might be different from other mathematics topics because probability topics do not necessarily begin with practical activities. In many mathematical topics, such practical activities provide a common basis for development of spoken, pictorial and written communication. They support the development of imagery, an important aspect of mathematics teaching (Pendlington, 1999). In probability lessons, primary teachers and children often use probabilistic language through reference to 'everyday' experiences rather than experimental situations in the first instance. A set of assumptions therefore exists about the sense that children make of these references, which might not be common to all. My analysis of the teacher and children's use of contextual references has highlighted the difficulties in using everyday contexts in order to make mathematics more meaningful for the children. Vygotsky (1987) emphasised the importance of children making sense of words in context, and raised issues about transference to other contexts also identified by Tirosh (1990). My research also raises issues about such transference of

meaning across different contexts when teaching probability, and from the pilot study also, when teaching fractions.

Although I did not set out to research the need for subject specialist language to communicate mathematics effectively, my research identified a need to ensure that teachers use such language in a way that is conceptually correct for mathematical understanding. Halliday (1978) described the 'social semiotic' as the context in which a person learns to mean, and where all subsequent meaning will take place. The classroom is essentially a semiotic situation. The influences on the development of meaning in such a situation are varied and Ernest (1997) describes semiotics as drawing on linguistic, cognitive, philosophical, historical, social, cultural and mathematical perspectives. To consider why children develop unclear meanings in such a situation I have taken account of some of these influences. In the previous section, I wrote about the need to develop 'precision in use' of mathematical language in relation to mathematical concepts. This takes account of the linguistic and mathematical perspectives in particular. I have also discussed the effects of 'everyday contexts' on the development of a shared understanding of meaning, and thus taken into account the social and cultural effects. Historically the requirement to develop a specialist subject language has been at the centre of debate in mathematics education research. Some such as Pimm (1997) and Pirie and Schwarzenberger (1988) do not believe that such use is essential to demonstrate mathematical understanding, but others such as Bennett (1996), and Barnes and Sheeran (1992) have identified that specialist language use is essential if children are to take part effectively in mathematics lessons. However, they all agree that teaching and learning a mathematical language is a complex task, as I have discussed in Chapter 2, and identified through my research. It certainly seems from my research that teachers' must strive to ensure children do not develop unclear meanings of mathematical words, such as those described by Fischbein (1991) and Steinbring (1991). I have already presented my recommendations in the previous section of this chapter, and these add to the argument for developing a secure subject



specialist language as part of the process of communicating and learning mathematics.

In order to teach children a secure knowledge of mathematical words and phrases teachers need to employ a range of communicative strategies and understand the nature of 'precision' proposed by the initial teacher training documentation from the Department for Education and Employment [DfEE] (Teacher Training Agency [TTA],1998). I have previously provided brief discussion the nature of precision in mathematics teaching and learning. I do not believe that it means exact definitions and statements, but that the concept of precision requires a flexibility that allows us to think of the notion of 'precision in use'. Thus the 'precision' that is required is directly related to the context in which the mathematical words or phrases are used. In Chapter 4, I identified that teachers do not generally introduce British primary school children to the use of numerical data in order to make predictions of probable outcomes for everyday events. The primary school curriculum does not specify use of data to make predictions in this way. Thus, the children in my research predicted that someone might have an even chance of living or dying, based on the idea that a person can either be dead or alive i.e. two possible outcomes. Here, I would claim that they have used 'even chance' in an imprecise way. 'Precision in use' for this particular 'contextual reference', requires them to have data by which they can justify their prediction.

Vygotsky (1987) and Lee (1985) wrote about children having spontaneous concepts of the meaning of a word that were recontextualised by new experiences and gradually became decontextualised. We might consider these as different levels of 'precision' because in each situation, at each stage of development and recontextualisation, the child is using the word in a precise way for their purpose. When a child uses 'even chance' instead of 'equal chance' in a sentence the meaning is precise in the sense that this is what the child understands by 'even chance' at that point. We can argue that it lacks precision because 'equal chance' is a phrase that includes any event with equally likely outcomes, while 'even chance' describes an event with

two equally likely outcomes. Thus, opportunities to recontextualise the meaning of 'even chance' through dialogue are necessary. I have discussed the implications for teaching and learning in the previous section of this chapter, highlighting the need for a variety of means of communication to support the development of a shared understanding of meaning. Each of the communicative events discussed provides an opportunity for recontextualisation to occur. Mason (1999) considered the role of labels in mathematics education, as I have previously discussed in Chapter 2. He suggested that there are times when we need a mathematical word or term, as a label, for precision in mathematical thinking. The word or phrase can conjure up images and ideas related to the concept that it represents and in this way 'triggers' mathematical thought. I consider that my notion of 'precision in use' relates well to Mason's ideas and it adds to the arguments about the relationship between mathematical conceptual development and linguistic development. However, I have also discussed in the previous section, the dangers of using mathematical words as labels without awareness of their meaning in context e.g. labels on the probability scale.

### **Evaluation of my research methods**

I adhered to the proposed methods, except that the narrative system for collecting information during lesson activity was inadequate. As in the pilot, there was still the problem of deciding which group to focus on, and the possibility of missing something elsewhere in the classroom. This resulted in records of some specific instances of dialogue, but mostly I made general observations. These indicated a lack of dialogue about the content of the lesson and more about the procedures of doing the activity. Teacher-pupil dialogue and pupil-pupil dialogue showed the same characteristic in this respect. I therefore focused on the whole-class interactions with the teacher when analysing the lessons. All other data collected on video was generally of good quality and showed clearly the links between gestures, or recording with pencil and paper, and the children's dialogue.

I collected too much video material in a short space of time. In retrospect it would have been useful to have video taped one lesson, transcribed, and

viewed it with the teacher before video taping another lesson. However, I was limited by the amount of time that the teacher was able to give within the school, and as a part-time researcher, the constraints of my own full-time employment. The amount of data collected made the task of transcribing seem enormous. Initially I tried to add all information to the transcript simultaneously. I transcribed every part of each lesson tape and the first three pairs of children during structured interventions. I transcribed everything as spoken, as each *'erm'* the children spoke conveyed the hesitant nature of some of their utterances. This was very time consuming. It was tempting to reduce a long-winded explanation to a shorter sentence that conveyed the same message, but in making such changes, I would falsify the exact nature of the transcript. Thus, the transcript is as true to reality as it can be. During transcription, I inevitably became interested in the dialogue and formulated ideas for analysis, to develop clearer focus. The quantity of materials collected became overwhelming and therefore it became necessary to maintain a clear focus on the research questions to make the amount of transcription manageable. My focus became the term *'even chance'* and so the remaining five transcriptions only contained references to *'even chance'*. In addition, it was easier to transcribe dialogue first, before revisiting the tapes to add observations. Transcription of tapes inevitably lost some of the meaning as identified by Pirie (1991) because the written text is a difficult medium in which to convey the particular nuances that are evident on tape. However, revisiting the tapes again to add comments helped in the analysis.

The structured interventions were variable in quality. The situation was not ideal, on the corridor near the stairs where children often passed on their way to other lessons. The presence of the camera affected children in different ways. Two were rather *'theatrical'* in their communications with their partners, while others were shy. In the pilot, I had been in the quiet library and had the camera further away. This time the children were very aware of the camera's close proximity although it was to one side. The opportunity to discuss mathematical language with another child was clearly a new experience of them all. Their responses reflected a variety of influences ranging from the external physical situation with a video camera

as a distraction, to the internal qualities such as personality. Asking children to explain how they made sense of a mathematical word without any physical aids was a challenging prospect, and so my intervention became inevitable. All of the above constitute part of the 'context of situation' (Edwards and Westgate, 1994). My interventions occurred more often than I wanted. I also felt that some of my responses also generated misunderstandings and left children following an inaccurate mathematical line of thought. This was the result of trying not to affect their thoughts by leading them into particular ways of thinking. I tried to be positive and encourage their responses, but children might interpret such encouragement as confirming their ideas as correct. This was a dilemma as I was attempting to discover what children were thinking. If I had not intervened then some children would have said very little. Intervention also helped to clarify misunderstandings as described in Chapter 3. My comments and questions to children were bound to have an effect. To counteract this effect I considered it in my analysis where appropriate.

The process of analysis began as I was watching and transcribing tapes. I began to make hypotheses about trends, and my comments reflect this. My initial analysis consisted of my own thoughts about the events. After this, I looked back at the literature review to clarify links and identify other research that agreed, or disagreed with my ideas. I then focused on strengthening the links between lessons, lessons and structured interventions and between different pairs of children. This seems to have been an effective way to work as it promoted a revisiting of tapes and transcripts to refine and develop ideas. I also looked back at previously read literature and searched for new material to develop ideas that the analysis identified as important features of communication. In this way, some sections of dialogue were closely analysed for their contribution to answering the research questions. It was difficult to retain the focus as other interesting findings occur.

### **Future research directions**

This chapter has drawn together practical classroom issues with theoretical developments in order to provide an overview of the contribution of my

research to both practice and theory. A small scale project looking in depth at communication processes in the classroom can provide a lot of detailed information that adds to, and identifies a range of key areas of concern in mathematics education. Further research into key elements identified as important on developing and maintaining a shared understanding of meaning in mathematics classrooms is necessary. The teacher had difficulty in ensuring a shared understanding of meaning, for a variety of reasons, and this has implications for the use of 'whole-class direct interactive teaching' recommended by the NNS (DfEE, 1999a). We need research into the best ways to support the professional development of teachers and initial teacher trainees in *effectively managing the development of a shared understanding of meaning of mathematical language*. In particular, such professional development must be orientated towards the inclusion of effective peer discussion, and teacher intervention techniques. In my recent experience as a numeracy consultant, current national training mechanisms are inadequate for providing teachers with such skills. My research also highlighted the importance of gesture as a communication process that some children relied on heavily. I have suggested previously that children ought to be encouraged to use gesture, and pictorial representations, when explaining their ideas. This is another line of future research. I would be particularly interested to explore whether encouraging children to develop ideas through gesture, or other means, will eventually help them develop the skills to enter educated discourse though words, in a more abstract way.

A strong element of my research was exploring the use of contextual references in dialogue and their influence on the development of meaning. My initial ideas about contextual references causing recipients to have different interpretations of events stemmed from a range of anecdotal evidence over a period of years, and the pilot study into fractional language strengthened these ideas. During the final study, I gathered a significant amount of evidence that such references do affect the development of meaning. In addition to affecting a shared understanding of meaning, I also identified that they affected the mathematical sense that a word or phrase had and could lead to poor conceptual development when associated with

other communication strategies such as gestures that also implied a different meaning. My initial hypothesis that such contextual references came from imagined situations, in which the speaker perceived the mathematical word applying, seems justified. A particular example was the reference to an 'even challenge' in football, and pictorial evidence shows that one child had an image of 'even chance' as an 'equal distance'. This subsequently led into the idea of balance, evenness, and equality shown in the discussion, drawings and gestures of one pair of children. Further research into this complex issue of how children perceive such references in relation to understanding the meaning of mathematical language would continue the debate.

Finally, I return to the notion of 'precision' because I believe that we should translate this as 'precision in use' rather than 'a correctly stated definition'. I can link each element of my research to the idea that we can use mathematical language in precise ways. Mathematics educators need to explore the meaning of 'correct use of mathematical language' in relation to children learning both the language and the mathematical concepts. Linguistic research discussed in Chapter 2 focused on children needing a specialist subject language in order to take full part in the lesson. While constructivist research into mathematics education, also discussed in Chapter 2, focused on children explaining mathematics in their own ways. Neither strand explored the link between linguistic and mathematical conceptual development so perhaps this is a further area for research in relation to the notion of 'precision'. Although I did not set out to consider mathematical conceptual development my research has highlighted that poor development of ideas about probabilistic language is likely to be associated with poor conceptual development e.g. if a child views 'even chance' as a balance, then it may affect further conceptual development when moving on to the working with fractional quantities as predictions. My particular concern here is that the approach we take towards teaching probabilistic language in primary school might not be the best preparation for secondary school probabilistic concepts. Again, I would suggest that this cross phase issue is a valid concern worthy of further research.

In conclusion, I believe that my research has raised important issues about communication processes and the development of a shared understanding of meaning in mathematics classrooms. It is challenging to attempt to combine different theoretical perspectives, but essential when focusing on classroom communication processes. I also wish to endorse the use of methods that focus in depth, on a small number of subjects, as an appropriate method for educational research projects. In particular, my methods are entirely appropriate for a teacher/researcher taking a sabbatical in order to inform their own and their colleagues' professional development. This might be one method of improving teachers' skills in managing the development of a shared understanding of meaning in the classroom.

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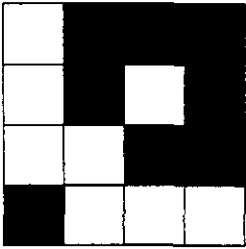
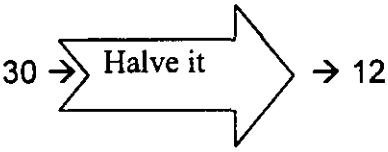









# **Appendices**

# Appendix 1: Pilot study record sheet (originally A3)

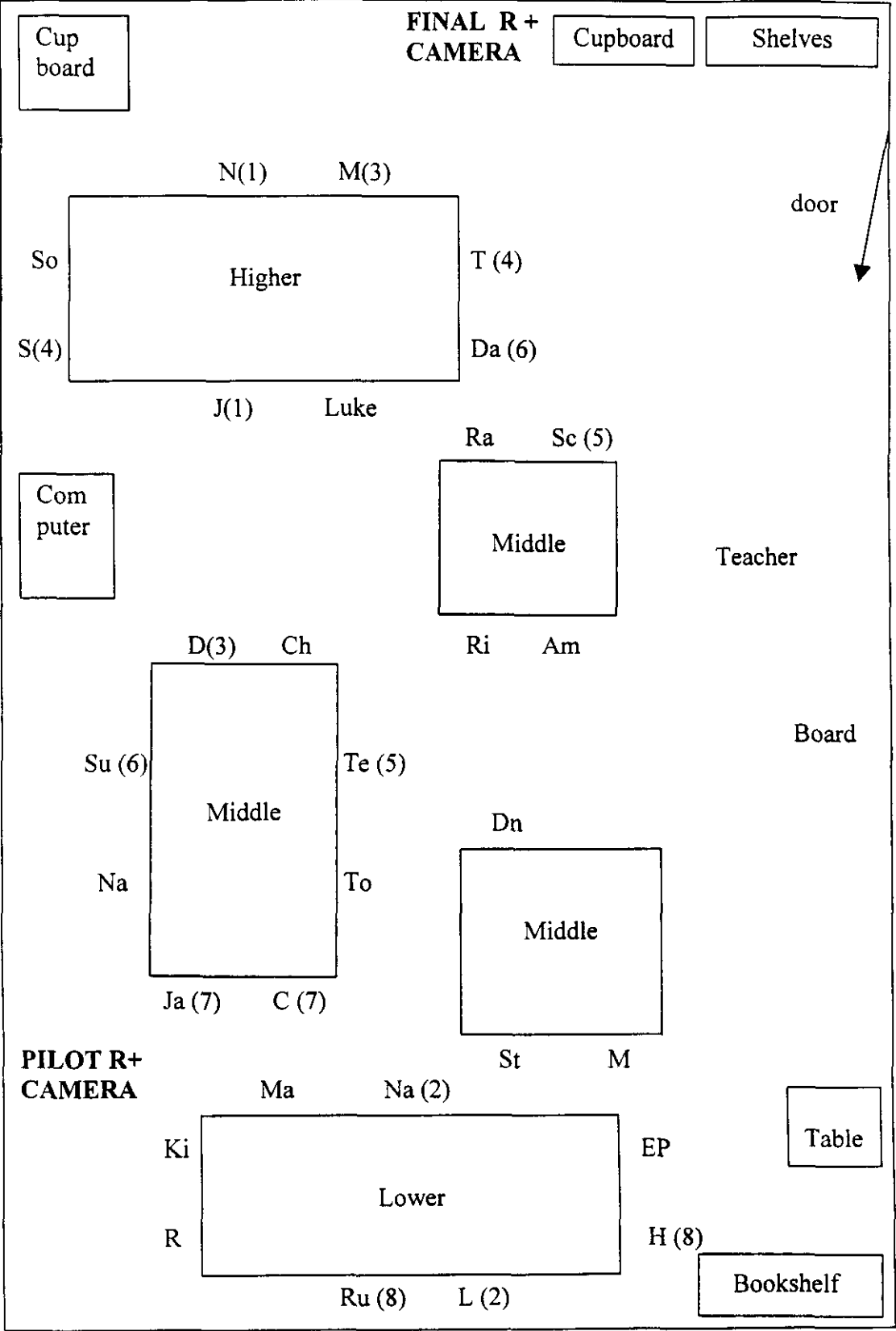
E 24/11/97      Name \_\_\_\_\_      Age ( ) \_\_\_\_\_      M/E' \_\_\_\_\_      Time 10-10.30

WORD	True or false?	description	picture	symbol	understand	read
① half		A half of a shape is when you cut something into two equal pieces		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
② fraction		A fraction is something like if you split a pizza into 4 equal pieces that's a fraction and there are loads of different numbers		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
③ denominator		A denominator is a number like if the top is the numerator because it tells you like how many parts I have been shaded		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
④ three sixths		Three sixths can be three sixths of a pizza has been cut into 6 equal pieces and three are left behind the decimal place is like + 1/2		$\frac{3}{6}$	$\frac{3}{6}$	$\frac{3}{6}$
⑤ decimal place		A decimal place is like the 1/2 behind the decimal point		$\frac{3}{10}$	$\frac{3}{10}$	$\frac{3}{10}$

## Appendix 2: Examples of pilot study 'true or false' cards

<p>0 <span style="margin-left: 100px;">↓</span> 10</p> <p>The place marked by an arrow is half way along the line.</p> <p>True or false?</p>	 <p>Half of this square is shaded.</p> <p>True or false?</p>
<p>I have a necklace with 50 beads on it. 25 beads are red. The rest are yellow.</p> <p>Half the beads are yellow.</p> <p>True or false?</p>	<p>If you put the number 30 into a 'halving' machine, the number 12 will come out at the other side.</p>  <p>True or false?</p>
<p>a)    = <math>\frac{6}{6}</math></p> <p>b)    = 3</p> <p>c)    = six halves</p> <p>All of these answers are correct.</p> <p>True or false?</p>	

**Appendix 3: Classroom plan**





#### **Appendix 4: Final transcript format example of sections used for the analysis - Lesson 2 'Even chance' 12/5/98**

No. = utterance number in this selection for analysis, purely for cross-referencing in the text.  
 Dialogue = utterances from one participant at a time.  
 Observation = events seen during the transcription of tapes  
 Comments = my notes, questions and developing ideas about the dialogue and observations.

##### a) The start of the lesson

No.	Dialogue	Observation	Comments
1	Teacher: What do you think 'even chance' might mean? Think about what the word 'even' means, and the word 'chance'. T?	A few children put hands up. Teacher scans room	Focus on one term. Not explicitly related to yesterday's terms.
2	Tim: Well, say (...old) you've got an 'even chance' of living and you've got a chance, you've got half a chance that you'll die and you've got half a chance that you'll live.		Child sets it into a context to which he can relate. Getting into the complex issues again.
3	Teacher: Super. Half a ch.... Tim has given the scenario of living and he says you've got an 'even chance' of living, which also means you've got an 'even chance' of not living. Right he's used half-and-half...Anything else? Jake?		Teacher clarifies what the child means to the class, but does not question whether it is a reasonable assumption to make. It sounds as if it is a fact, rather than an opinion or

			judgement.
4	Jake: The tackle was 'fifty-fifty'.		Child chooses context.
5	Teacher: The tackle was 'fifty-fifty'. Explain that. There's non-footballers in the class.		Child asked to explain.
6	Jake: It means that you've made a challenge.		
7	Teacher: An even challenge, yep. Anyone else? No,...Right! I've got some cubes, alright? I'm going to put them in...it's a little memory game. You've got to remember three red cubes and two blue cubes. How many red Naomi?	Teacher picks up a box of cubes	Teacher sets context. Numerical. Teacher seems to change his mind mid sentence. Has happened before. Closed question
8	Naomi: Three		
9	Teacher: How many blue David?		Closed
10	David: Two.		
11	Teacher: Right, do you think...hands up if you think if I said, 'pick a cube', it would be ' <i>even chance</i> '... Hands up if you think it would be ' <i>uneven chance</i> '... <i>OK</i> <b>Richie</b> , explain why.	No hands up  All hands up Barely audible	'Right' signals that the teacher is ready to start, not that David is correct. Why does he use 'uneven'?
12	Richie: Because there's more red than blue.		Numerical quantities make comparisons and judgements

			easier
--	--	--	--------

- b) The teacher begins to explain the different activities for the children to complete. Approximately 9 minutes into the lesson.

17	Teacher: ... First one..... It's... with a dice... alright? How could use the dice to show 'even chance'? How could the shake of a dice show 'even chance'? Stuart.	Teacher holds up a die.	
18	Stuart: It's like there's one of each number.		
19	Teacher: There is one of each number right. How else could we do it is so that there is only two outcomes? Before we had; I know it is difficult to think; there was two outcomes. Nita?	Points to the board	Teacher does not explicitly correct the child's misconception, which might leave other children thinking Stuart's answer is 'even chance'.
20	Nita: Even and odd.		
21	Teacher: Even and odd. That's what we're going to do, alright? We are going to see if when we shake the dice it's an even number or an odd number. Tess, which numbers are on the dice? What numbers.....		Teacher has led the talk to a more accurate description of 'even chance'
22	Tess: One two three four five.	Teacher nods, supportive facial expression and gestures	

- c) The teacher explains the final activity using a plastic cup.  
Approximately 12 minutes into the lesson

30	Teacher: Ten, right. <b>We'll</b> just have one more activity, alright? And I want you just to put your hands up and tell me if you think this is a good chance or not. Alright? I've got a cup. All you are going to do is you're going to throw it. Right. <b>Not</b> a silly throw, just a little throw, but each time you are going to do it from a similar height, why?... Stuart.	He is holding a plastic cup Demonstrates with gesture, without actually letting go of the cup.	Using voice to keep attention.  Voice raised to control The gesture implies an upward throw
31	Stuart: It's got to be same chance.		Meaning equal chance or 'even chance'? Or fair?
32	Teacher: What does it make it though? Same chance.		
33	Stuart: Fair.		
34	Teacher: Fair ..... test. A fair test. Alright?...Sophie?	Looks at Stuart to elicit response. So's hand is up	Wants Stuart to say 'fair test'
35	Sophie: Sir it won't be a fair test because the open end will come down more easily.		Confident enough to challenge.
36	Teacher: No that doesn't mean it won't be a fair test. That's depending on whether it's an 'even chance' or not. OK.  Sophie: I'm (inaudible) from	Directly to So Gesture – hand from side to side. Holding cup with rim down	He accepts the challenge is valid and explains why she is incorrect in her assumption. The gesture suggests

	here. What three ways could it fall? What ways could it possibly land? C?		balance
--	---	--	---------

- d) After the practical work, there is discussion of the results. Approximately 27 minutes into the lesson.

55	Teacher: Right, Liam, what did you find please?		
56	Liam: We got ten red and ten blue.		
57	Teacher: Ten red... What can you say about that then Liam?...mathematically.	Writes on the board	
58	Liam: 'Even chance'.		
59	Teacher: 'Even chance'...what else could you say? Jake.		
60	Jake: Fair test		
61	Teacher: Yeah, I was thinking of something else. Sophie?		Guess the teacher's answer?
62	Sophie: Fifty- fifty.		
63	Teacher: Fifty-fifty, I was thinking...you got what you expected didn't you?...Ki give us your results.	Small gesture with one hand rocking	

## Appendix 5: Summary of teacher's comments on Lesson 1 – Probability 11/5/98

*(The summary of the teacher's comments is in italics)*

Q 1. Did the children understand what the word 'probability' meant?

- *No. I gave no explanation.*
- *I just said we were revising 'probability'.*
- *I didn't explain it at all.*

Q 2. Were you aware that you used the phrase 'numbers between 1 and 49'?

- *Yes – what's wrong with that?*
- *I clarified by asking for examples.*
- *Should I have said 'whole numbers' or something?*

Q 3. Are there any instances of words or phrases that you think you used badly?

Q 4. Is there any terminology that you could have used better?

- *No specific answers to these two questions*
- *The teacher felt that his response to Q5 answered these questions too.*

Q 5. If you did the first lesson again is there anything you would change.

- *I would be better prepared for the ambiguity!*
- *Provide given examples to categorise, before asking for contributions*
- *Use clear cut, less ambiguous examples, for better understanding*
- *I didn't realise how difficult it was to do!*

Q 6. You tend to use closed questions more than open questions. Can you explain why?

- *I didn't realise that I did.*
- *I thought I began with open ones then asked closed ones if children were having difficulties.*
- *Maybe it's because I know what answer I am expecting, what I want to hear, and I am guiding them towards that.*
- *I shouldn't really do that I suppose.*

Q 7. When the children were providing examples, in the first lesson, were you conscious that you began to provide explanations for their examples? Could you have managed this aspect of the lesson in a different way?

- *I think I was trying to justify them in my own head, but out loud!*
- *As it wasn't clear, I was sounding things out by trying to make sense.*
- *I was trying to explain to the others.*
- *I should have asked them to explain their choices themselves.*
- *They did this for the textbook exercises and it worked well.*

Q 8. What do you think is the best way to secure a child's understanding of the language?

- *Give examples from real life situations*
- *Ask them to explain in words, in a mathematical sentence*
- *Ask for situations and contexts to support this*

Q 9. What is your opinion of the textbook material you used after observing the children's response to it?

- *It was good to show how difficult it was.*
- *I was unprepared.*
- *I skimmed through it because it looked straightforward, how wrong I was.*
- *Some examples were better than others, it got us all thinking*
- *Page 123 was better when children had to identify which was most or least likely.*
- *Also the activity based scenarios e.g. the badge question*

Q 10. Comment on the children's responses to your whole-class parts of the lessons.

- *Some were silly as they considered example which were personal to them (perhaps they did not intend to be daft.)*
- *Children were keen to contribute, and to disagree with reasons.*
- *The confusion earlier in the lesson made children more careful when giving examples at the end.*
- *Some began to give extra justification to stop others arguing.*
- *They began to think of events in the past for 'certain'*

Q 11. Many of the words used in probability scales are everyday words rather than mathematical terms. What problem does this cause?

- *'Likely' depends on personal experiences/opinions, and cannot easily be defined.*
- *It depends on the time of year, and other circumstances*
- *Overall, the lesson got the children thinking quite deeply, although it was still a nightmare (even looking at it now!)*
- *Language seemed difficult to explain and understand*
- *Children used 'likely' as means of not committing themselves to situations, and to avoid certain and impossible.*



## Appendix 6: Summary of teachers comment on Lesson 2 – ‘Even chance’ 12/5/98

*(The summary of the teacher's comments is in italics)*

Q 1. What is the main difference between this lesson and the first one?

- *It involved hands on activities with measurable outcomes. The children could ‘see’ the probability.*
- *More open-ended questions, e.g. “How could dice show ‘even chance’?” “What else shows ‘even chance’?”*
- *Clear explanations resulted in children knowing the expectations.*
- *Clarifying the meaning of ‘even chance’ from the beginning.*
- *Not certain whether reference to ‘uneven’ chance was a good idea.*
- *Children made good use of language introduced the previous day e.g. unlikely*
- *A good summary session – the children were interested in the results.*

Q 2. How successful was this lesson in teaching the terminology, including the word ‘probability’

- *Children had good understanding of ‘even’ and ‘uneven’ chance by the end of the lesson – from discussion and practical experience*
- *They knew how to alter a situation from that of ‘even chance’ to one which was not, using the cubes in the bag.*
- *Children understood the words ‘expected’ and ‘outcome’ and understood when the unexpected happened.*

Q 3. Were there any differences in achievement in the class?

- *Not at this stage, they all understood the activities.*
- *Some of them even ‘fixed’ their results to ensure they showed ‘even chance’.*

Q 4. What are the causes for these differences? Is it to do with mathematical ability or language?

- *The language was understood; any differences are due to mathematical ability i.e. ability to calculate the expected outcomes.*

## **Appendix 7: Summary of teacher's comments on Lesson 3 – The probability scale 13/5/98**

*(The summary of the teacher's comments is in italics)*

Q 1. What was the benefit of using this approach in relation to the children learning how to use the language involved?

- *Varying degrees of probability*
- *Different ways of stating outcomes e.g. fairly, quite, maybe, 'even chance', when interpreting the positions between likely and unlikely*
- *Children progressed from lesson 1*
- *They used 'nearly certain' and 'nearly impossible', in a way which was hard to express in lesson 1-children wanted to be exactly 'certain' and 'impossible' in lesson 1*

Q 2. Was it useful to use the analogy of the 'time line'?

- *Children could record and see whether something was nearer to 'likely' or 'unlikely'*
- *They could generate ideas dependent on their own lifestyle*
- *If they couldn't distinguish between 'certain' and 'very likely' they could place it between*
- *In lesson 1 children felt they could not change from their original suggestions, but here they could*

Q 3. What difficulties are there in relation to the children completing their own individual probability scales?

- *Because of the dependence upon individual circumstances, e.g. 'get wet' depends on the weather.*
- *Higher ability understood this and identified some of the factors*
- *Lower ability found it difficult and weren't really convinced – this was due to language more than mathematical ability*



Q 4. How easy/difficult is it to check the children's understanding?

- *From the time lines I could see what the children perceived as being 'likely' or 'unlikely' etc. especially when using their own examples.*

## Appendix 8: Writing and drawing to support oral explanations

Notes:

The child identified is the one that explained 'even chance'. In Pair 4 both children recorded as indicated.

Pair	Written explanation or pictures
1. Nita	Even chance is where you have maybe a box of counters and if the same amount are blue as red it is 'even chance'. If you have more of one colour, it isn't 'even chance'.
2. Lee	$\frac{1}{2} \frac{1}{2}$
3. Diane	Even chance means if you have six numbers you have an even chance of getting an odd or an even number. 0 0 0   0 0 0 even chance
4. Tim and Sally	$100 \div 2 = 50$ (T) $\oplus \oplus \odot \oplus$ $\odot \odot \oplus \odot$ (T) 1 2 1 3 + + o + (T) $o^2 o^3 +^4 o^4$ $\Delta \Delta \bullet$ $\bullet \bullet \Delta$ 3 3 (S) $\Delta \bullet$ 5,24,42,33,17,26 (S) <div style="text-align: center;">10</div>
5. Stuart	
6. Sue	 This is even chance because there is the same amount of each colour

Appendix 8 continued:

7. Jan	<div><div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div><div><div></div><div></div></div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div></div><div><div><div><div></div><div></div></div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div></div></div></div></div>
8. Ryan	